Name: ______________________ Date: __________

CONTENT 4.1

4.1 Speed

READ

To determine the speed of an object, you need to know the distance traveled and the time taken to travel that distance. If you know the speed, you can determine the distance traveled or the time it took—you just rearrange the formula for speed, $v = d/t$. For example,

Equation...	Gives you...	If you know...
$v = d/t$	speed	distance and time
$d = v \times t$	distance	speed and time
$t = d/v$	time	distance and speed

Use the SI system to solve the practice problems unless you are asked to write the answer using the English system of measurement. As you solve the problems, include all units and cancel appropriately.

EXAMPLE

Example 1: What is the speed of a cheetah that travels 112.0 meters in 4.0 seconds?

Looking for	Solution
Speed of the cheetah.	$\text{speed} = \frac{d}{t} = \frac{112.0\text{ m}}{4.0\text{ s}} = \frac{28\text{ m}}{\text{s}}$ The speed of the cheetah is 28 meters per second.
Given Distance = 112.0 meters Time = 4.0 seconds	
Relationship $\text{speed} = \frac{d}{t}$	

Example 2: There are 1,609 meters in one mile. What is this cheetah's speed in miles/hour?

Looking for	Solution
Speed of the cheetah in miles per hour.	$\frac{28\text{ m}}{\text{s}} \times \frac{1\text{ mile}}{1{,}609\text{ m}} \times \frac{3{,}600\text{ s}}{1\text{ hour}} = \frac{63\text{ miles}}{\text{hour}}$ The speed of the cheetah in miles per hour is 63 mph.
Given Speed = 28 m/s (from solution to Example 1)	
Relationships $\text{speed} = \frac{d}{t}$ and 1, 609 meters = 1 mile	

PRACTICE

1. A bicyclist travels 60.0 kilometers in 3.5 hours. What is the cyclist's average speed?

Looking for	Solution
Given	
Relationships	

2. What is the average speed of a car that traveled 300.0 miles in 5.5 hours?
3. How much time would it take for the sound of thunder to travel 1,500 meters if sound travels at a speed of 330 m/s?
4. How much time would it take for an airplane to reach its destination if it traveled at an average speed of 790 kilometers/hour for a distance of 4,700 kilometers? What is the airplane's speed in miles/ hour?
5. How far can a person run in 15 minutes if he or she runs at an average speed of 16 km/hr? (HINT: Remember to convert minutes to hours.)
6. In problem 5, what is the runner's distance traveled in miles?
7. A snail can move approximately 0.30 meters per minute. How many meters can the snail cover in 15 minutes?
8. You know that there are 1,609 meters in a mile. The number of feet in a mile is 5,280. Use these equalities to answer the following problems:
   a. How many centimeters equals one inch?
   b. What is the speed of the snail in problem 7 in inches per minute?
9. Calculate the average speed (in km/h) of a car stuck in traffic that drives 12 kilometers in 2 hours.
10. How long would it take you to swim across a lake that is 900 meters across if you swim at 1.5 m/s?
    a. What is the answer in seconds?
    b. What is the answer in minutes?
11. How far will a you travel if you run for 10. minutes at 2.0 m/s?
12. You have trained all year for a marathon. In your first attempt to run a marathon, you decide that you want to complete this 26.2-mile race in 4.5 hours.
    a. What is the length of a marathon in kilometers (1 mile = 1.6 kilometers)?
    b. What would your average speed have to be to complete the race in 4.5 hours? Give your answer in kilometers per hour.

13. Suppose you are walking home after school. The distance from school to your home is five kilometers. On foot, you can get home in 25 minutes. However, if you rode a bicycle, you could get home in 10 minutes.
    a. What is your average speed while walking?
    b. What is your average speed while bicycling?
    c. How much faster you travel on your bicycle?
14. Suppose you ride your bicycle to the library traveling at 0.50 km/min. It takes you 25 minutes to get to the library. How far did you travel?
15. You ride your bike for a distance of 30 km. You travel at a speed of 0.75 km/ minute. How many minutes does this take?
16. A train travels 225 kilometers in 2.5 hours. What is the train's average speed?
17. An airplane travels 3,280 kilometers in 4.0 hours. What is the airplane's average speed?
18. A person in a kayak paddles down river at an average speed of 10. km/h. After 3.25 hours, how far has she traveled?
19. The same person in question 18 paddles upstream at an average speed of 4 km/h. How long would it take her to get back to her starting point?
20. An airplane travels from St. Louis, Missouri to Portland, Oregon in 4.33 hours. If the distance traveled is 2,742 kilometers, what is the airplane's average speed?
21. The airplane returns to St. Louis by the same route. Because the prevailing winds push the airplane along, the return trip takes only 3.75 hours. What is the average speed for this trip?
22. The airplane refuels in St. Louis and continues on to Boston. It travels at an average speed of 610 km/h. If the trip takes 2.75 hours, what is the flight distance between St. Louis and Boston?

Challenge Problems:

23. The speed of light is about 3.00×10^5 km/s. It takes approximately 1.28 seconds for light reflected from the moon to reach Earth. What is the average distance from Earth to the moon?
24. The average distance from the sun to Pluto is approximately 6.10×10^9 km. How long does it take light from the sun to reach Pluto? Use the speed of light from the previous question to help you.
25. Now, make up three speed problems of your own. Give the problems to a friend to solve and check their work.
    a. Make up a problem that involves solving for average speed.
    b. Make up a problem that involves solving for distance.
    c. Make up a problem that involves solving for time.

Name: ______________________________ Date: __________

4.1 Velocity

CONTENT 4.1

READ

Speed and velocity do not have the same meaning to scientists. Speed is a *scalar quantity*, which means it can be completely described by its magnitude (or size). The magnitude is given by a number an a unit. For example, an object's speed may be measured as 15 meters per second.

Velocity is a *vector quantity*. In order to measure a vector quantity, you must know the both its magnitude and direction. The velocity of an object is determined by measuring both the *speed* and *direction* in which an object is traveling.

- If the speed of an object changes, then its velocity also changes.
- If the direction in which an object is traveling changes, then its velocity changes.
- A change in either speed, direction, or both causes a change in velocity.

You can rearrange **v** = *d*/*t* to solve velocity problems the same way you solved speed problems earlier in this course. The boldfaced **v** is used to represent velocity as a vector quantity. The variables *d* and *t* are used for distance and time. **The velocity of an object in motion is equal to the distance it travels per unit of time in a given direction**.

EXAMPLES

Example 1: What is the velocity of a car that travels 100.0 meters, northeast in 4.65 seconds?

Looking for Velocity of the car.	**Solution** $\text{velocity} = \frac{d}{t} = \frac{100.0\text{ m}}{4.65\text{ s}} = \frac{21.5\text{ m}}{\text{s}}$ The velocity of the car is 21.5 meters per second, northeast.
Given Distance = 100.0 meters Time = 4.65 seconds	
Relationship $\text{velocity} = \frac{d}{t}$	

Example 2: A boat travels with a velocity equal to 14.0 meters per second, east in 5.15 seconds. What distance in meters does the boat travel?

Looking for Distance the boat travels.	**Solution** $\text{distance} = v \times t = \frac{14.0\text{ m}}{\text{s}} \times 5.15\text{ s} = 72.1\text{ m}$ The boat travels 72.1 meters.
Given Velocity = 14.0 meters per second, east Time = 5.15 seconds	
Relationship $\text{distance} = v \times t$	

PRACTICE

1. An airplane flies 525 kilometers north in 1.25 hours. What is the airplane's velocity?

Looking for	**Solution**
Given	
Relationship	

2. A soccer player kicks a ball 6.5 meters. How much time is needed for the ball to travel this distance if its velocity is 22 meters per second, south?

3. A cruise ship travels east across a river at 19.0 meters per minute. If the river is 4,250 meters wide, how long does it take for the ship to reach the other side?

4. Joaquin mows the lawn at his grandmother's home during the summer months. Joaquin measured the distance across his grandmother's lawn as 11.5 meters.
   a. If Joaquin mows one length across the lawn from east to west in 7.10 seconds, then what is the velocity of the lawnmower?
   b. Once he reaches the edge of the lawn, Joaquin turns the lawnmower around. He mows in the opposite direction but maintains the same speed. What is the velocity of the lawnmower?

5. A family drives 881 miles from Houston, Texas to Santa Fe, New Mexico for vacation. How long will it take the family to reach their destination if they travel at a velocity of 55.0 miles per hour, northwest?

6. A shopping cart is pushed 15.6 meters west across a parking lot in 5.2 seconds. What is the velocity of the shopping cart?

7. Katie and her best friend Liam play tennis every Saturday morning. When Katie serves the ball to Liam, it travels 9.5 meters south in 2.1 seconds.
   a. What is the velocity of the tennis ball?
   b. If the tennis ball travels at constant speed, what is its velocity when Liam returns Katie's serve?

8. A driver realizes that she is traveling in the wrong direction on a one-way street. She has already driven 350 meters at a velocity of 16 meters per second, east before deciding to make a U-turn. How long did it take for the driver to realize her error?

9. Juan's mother drives 7.25 miles southwest to her favorite shopping mall. What is the average velocity of her automobile if she arrives at the mall in 20. minutes?

10. A bus is traveling at 79.7 kilometers per hour east. How far does the bus travel 1.45 hours?

11. A girl scout troop hiked 5.8 kilometers southeast in 1.5 hours.What was the troop's velocity?

12. A volcanologist noted that a lahar rushed down a mountain at 32.2 kilometers per hour, south. How far did the mud flow in 17.5 minutes?

Name: ______________________ Date: __________

CONTENT 4.1

4.1 Position on the Coordinate Plane

READ

To describe any location in two dimensions, we use a grid called the **coordinate plane**. You can describe any **position** on the coordinate plane using two numbers called **coordinates**, which are shown in the form of (*x*, *y*). These coordinates are compared to a fixed reference point called the **origin**. The table below describes the *x* and *y* coordinates:

Coordinate	Which axis is it on?	Which is the positive direction?	Which is the negative direction?
x	horizontal, called the *x*-axis	right or east	left or west
y	vertical, called the *y*-axis	up or north	down or south

EXAMPLE

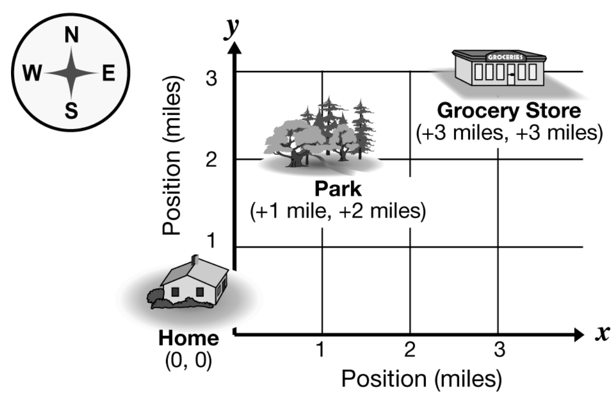
Your home is at the origin, and a park is located 2 miles north and 1 mile east of your home.

- Show your home and the park on a coordinate plane, and give the coordinates for each.
- After you go to the park, you drive 2 miles east and 1 mile north to the grocery store. What are the coordinates of the grocery store?

Solution

If your home is at the origin, it is given the coordinates (0, 0). By counting over 1 box from the origin in the positive *x*-direction and up 2 boxes in the positive *y*-direction, you can place the park on the coordinate plane. The park's coordinates are (+1 mile, +2 miles).

From the park, count over 2 more boxes in the positive *x*-direction and up one more 1 box in the positive *y*-direction to place the grocery store. That makes the grocery store's coordinates (+3 miles, +3 miles).



PRACTICE

1. You are given directions to a friend's house from your school. They read: "Go east one block, turn north and go 4 blocks, turn west and go 1 block, then go south for 2 blocks." Using your school as the origin, draw a map of these directions on a coordinate plane. What are the coordinates of your friend's house?

2. A dog starts chasing a squirrel at the origin of a coordinate plane. He runs 20 meters east, then 10 meters north and stops to scratch. Then he runs 10 meters west and 10 meters north, where the squirrel climbs a tree and gets away.

   a. Draw the coordinate plane and trace the path the dog took in chasing the squirrel.

   b. Show where the dog scratched and where the squirrel escaped, and give coordinates for each.

3. Does the order of the coordinates matter? Is the coordinate (2, 3) the same as the coordinate (3, 2)? Explain and draw your answer on a coordinate plane.

Name: ______________________ Date: __________

CONTENT 4.1

4.1 Vectors on a Map

READ

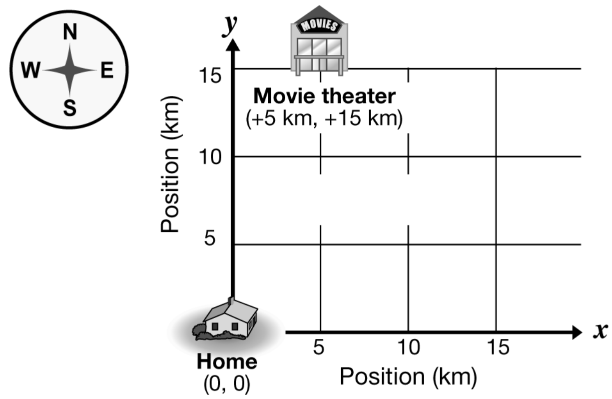
You have learned that velocity is a vector quantity—this means that when you talk about velocity, you must mention both speed and direction. You can use velocity vectors on a coordinate plane to help you figure out the position of a moving object at a certain point in time.

EXAMPLE

Your home is at the origin. From there you ride your bicycle to the movie theater. You ride 30. km/hr north for 0.50 hour, and then 20. km/hr east for 0.25 hours.

Show your home and the movie theater on a coordinate plane, and give the coordinates for each.

Solution:



If your home is at the origin, it is given the coordinates (0, 0). To find the position of the movie theater, you need to find the change in position. Use the relationship:

change in position = velocity × change in time

First change in position: +30. km/hr × 0.50 hr = 15 km NORTH

Second change in position: +20. km/hr × 0.25 = 5 km EAST

From home, travel north 15 km. Then turn and go east 5 km.

The coordinates of the movie theater are (+5 km, +15 km).

Note: Be careful to report the *x*-coordinate first. It does not matter which direction you traveled first. When reporting position, you always give the *x*- (east-west) coordinate first, then the *y*- (north-south) coordinate.

PRACTICE

1. Augustin and Edson are going to a baseball game. To get to the stadium, they travel east on the highway at 120. km/hr for 30. minutes. Then they turn onto the stadium parkway and travel south at 60. km/hr for 10. minutes. Assume their starting point is at the origin. What is the position of the stadium?
2. Destiny and Franijza are at the swimming pool. They decide to walk to the ice cream shop. They walk north at a pace of 6 km/hr for 20. minutes, and then east at the same pace for 10. minutes. If the swimming pool is at the origin (0, 0) what is the position of the ice cream shop?
3. After finishing their ice cream, the girls decide to go to Destiny's house. From the ice cream shop, they walk south at a pace of 4.0 km/hr for 15 minutes. What is the position of Destiny's house?
4. Draw a map showing the swimming pool at the origin (0, 0). Show the coordinates of the ice cream shop and Destiny's house.
5. **Challenge!** Make up your own velocity question. Your object (or traveler) should make at least one turn. Use at least two different speeds in your problem. Trade questions with a partner. Use a coordinate plane to help you solve the new question.

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4.2 Calculating Slope from a Graph

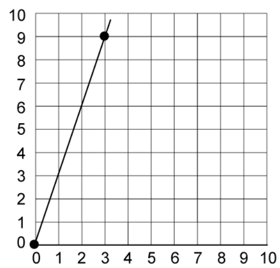
READ

To determine the slope of a line in a graph, first choose two points on the line. Then count how many steps up or down you must move to be on the same horizontal line as your second point. Multiply this number by the scale of your horizontal axis. For example, if your *x*-axis has a scale of 1 box = 20 cm, then multiply the number of boxes you counted by 20 cm.

Put the result along with the positive or negative sign as the top (numerator) of your slope ratio. Then count how many steps you must move right or left to land on your second point. Multiply the number of steps by the scale of your vertical axis. Place the results as the bottom (denominator) of your slope ratio. Then reduce the fraction of your ratio. This number is the slope of the line. Note: The letter *m* is used to represent slope in an equation.

EXAMPLES

A

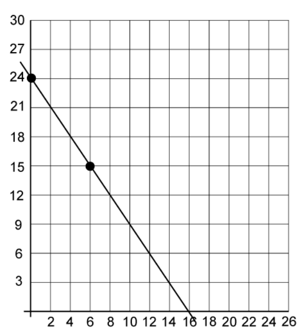


The chosen points for Example A are (0, 0) and (3, 9). There are many choices for this graph, but only one slope. If you have the point (0, 0), you should choose it as one of your points.

It takes 9 vertical steps to move from (0, 0) to (0, 9). Put a 9 in the numerator of your slope ratio (or subtract 9 – 0). Then count the number of steps to move from (0, 9) to (3, 9). This is your denominator of your slope ratio. Again, you can do this by subtraction (3 – 0).

$$m = \frac{9}{3} = \frac{3}{1}$$

B



The two points that have been chosen for Example B are (0, 24) and (6, 15). Be careful of the scales on each of the axes.

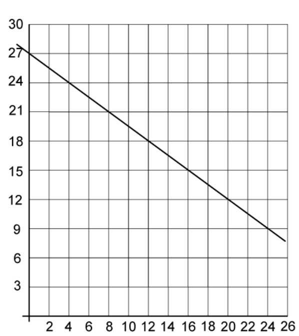
It takes 3 vertical steps to go from (0, 24) to (0, 15). But each of these steps has a scale of 3. So you put a –9 into the numerator of the slope ratio. It is *negative* because you are moving down from one point to the other. Then count the steps over to (6, 15). There are 3 steps but each counts for 2 so you put a 6 into the denominator of the slope ratio.

$$m = \frac{-9}{6} = \frac{-3}{2}$$

PRACTICE

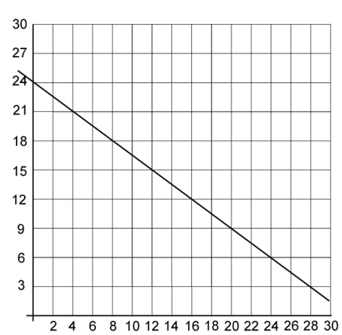
Find the slope of the line in each of the following graphs:

Graph #1:



m = ______

Graph #2:



m = ______

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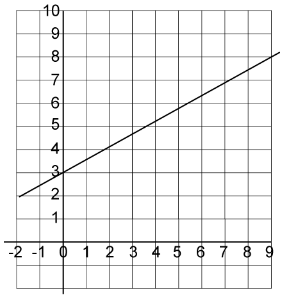
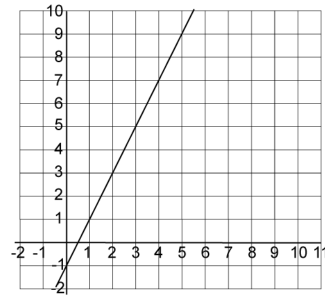
Graph #3:



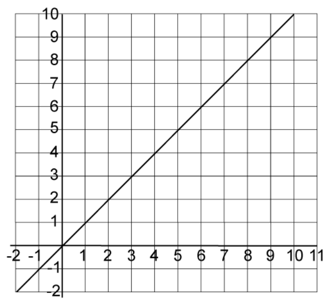
m =______

Graph #4:



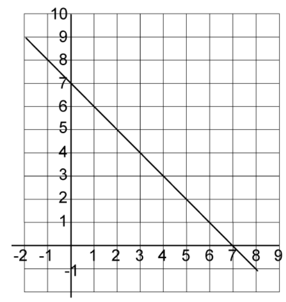
m =______

Graph #5:



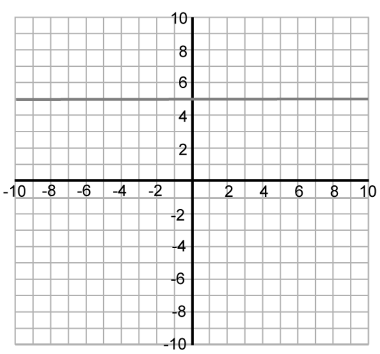
m =______

Graph #6:



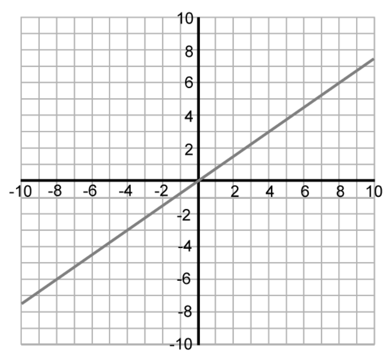
m =______

Graph #7:



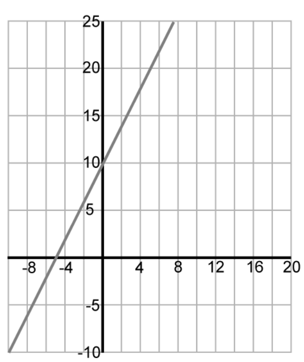
m = ______

Graph #8:



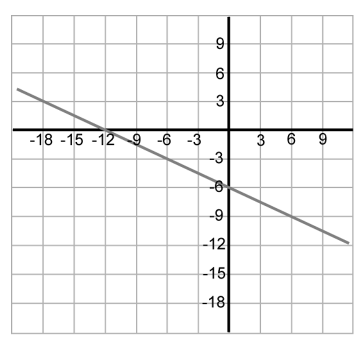
m = ______

Graph #9:



m = ______

Graph #10:



m = ______

Name: ______________________ Date: __________

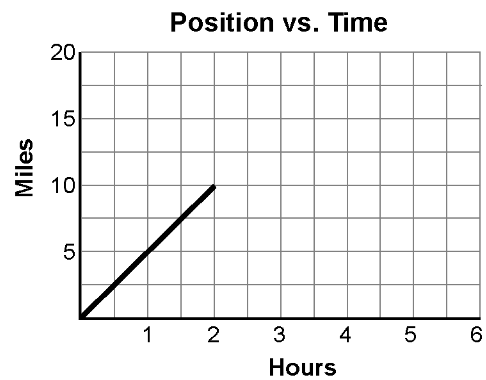
4.2 Analyzing Graphs of Motion With Numbers

CONTENT 4.2

READ

Speed can be calculated from position-time graphs and distance can be calculated from speed-time graphs. Both calculations rely on the familiar speed equation: $v = d/t$.

This graph shows position and time for a sailboat starting from its home port as it sailed to a distant island. By studying the line, you can see that the sailboat traveled 10 miles in 2 hours.



EXAMPLES

- **Calculating speed from a position-time graph**

The speed equation allows us to calculate that the boat's speed during this time was 5 miles per hour.

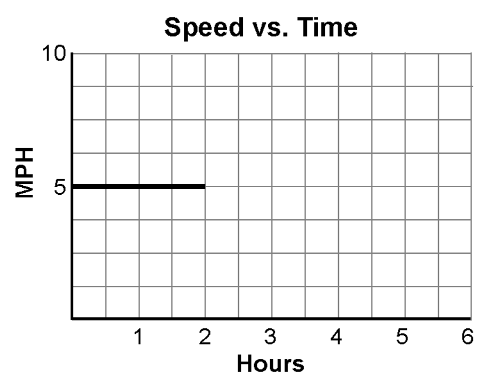
$$v = d/t$$

$$v = 10 \text{ miles} / 2 \text{ hours}$$

$$v = 5 \text{ miles/hour, read as 5 miles per hour}$$

This result can now be transferred to a speed-time graph. Remember that this speed was measured during the first two hours.

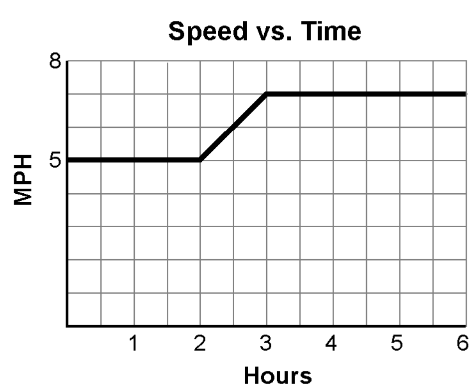
The line showing the boat's speed is horizontal because the speed was constant during the two-hour period.



- **Calculating distance from a speed-time graph**

Here is the speed-time graph of the same sailboat later in the voyage. Between the second and third hours, the wind freshened and the sailboat gradually increased its speed to 7 miles per hour. The speed remained 7 miles per hour to the end of the voyage.

How far did the sailboat go during the six-hour trip? We will first calculate the distance traveled between the third and sixth hours.



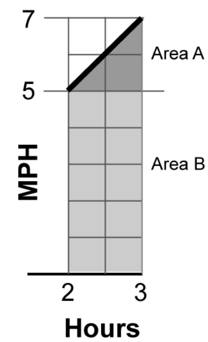
On a speed-time graph, distance is equal to the area between the baseline and the plotted line. You know that the area of a rectangle is found with the equation: $A = L \times W$. Similarly, multiplying the speed from the y-axis by the time on the x-axis produces distance. Notice how the labels cancel to produce miles:

$$\text{speed} \times \text{time} = \text{distance}$$

$$7 \text{ miles/hour} \times (6 \text{ hours} - 3 \text{ hours}) = \text{distance}$$
$$7 \text{ miles/}\cancel{\text{hour}} \times 3 \ \cancel{\text{hours}} = \text{distance} = 21 \text{ miles}$$

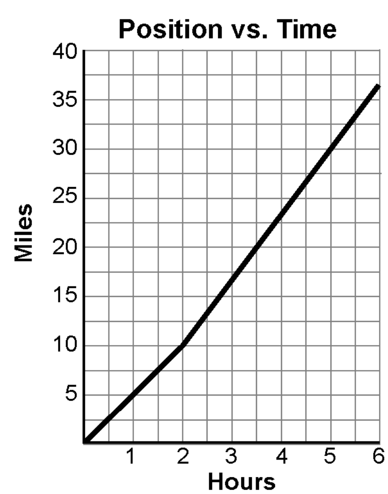
Now that we have seen how distance is calculated, we can consider the distance covered between hours 2 and 3.

The easiest way to visualize this problem is to think in geometric terms. Find the area of the triangle (Area A), then find the area of the rectangle (Area B), and add the two areas.



Area of triangle A Geometry formula	The area of a triangle is one-half the area of a rectangle. $\text{speed} \times \frac{\text{time}}{2} = \text{distance}$ $(7 \text{ miles/hour} - 5 \text{ miles/hour}) \times \frac{(3 \text{ hours} - 2 \text{ hours})}{2} = \text{distance} = 1 \text{ mile}$
Area of rectangle B Geometry formula	$\text{speed} \times \text{time} = \text{distance}$ $5 \text{ miles/hour} \times (3 \text{ hours} - 2 \text{ hours}) = \text{distance} = 5 \text{ miles}$
Add the two areas	$\text{Area A} + \text{Area B} = \text{distance}$ $1 \text{ miles} + 5 \text{ mile} = \text{distance} = 6 \text{ miles}$

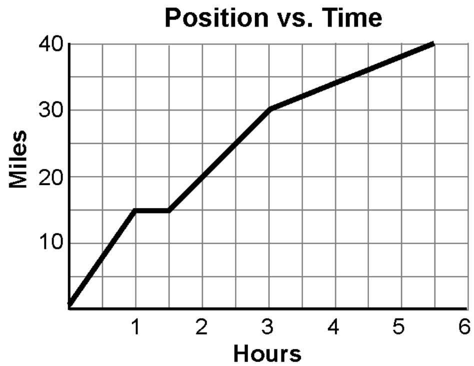
We can now take the distances found for both sections of the speed graph to complete our position-time graph:

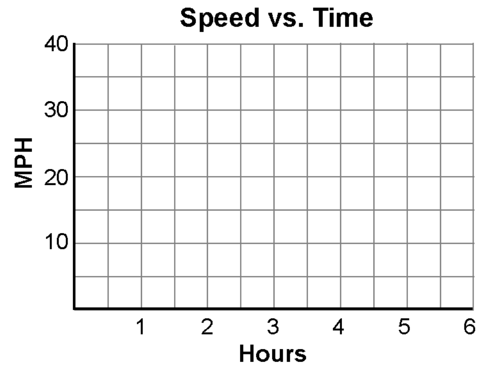


PRACTICE

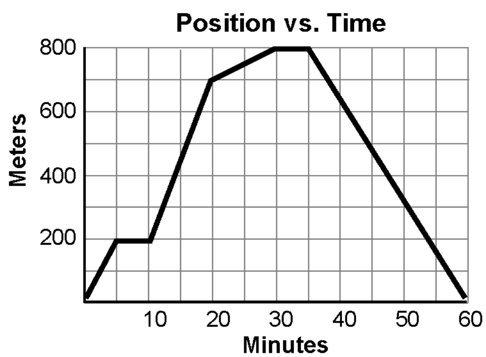
1. For each position-time graph, calculate and plot speed on the speed-time graph to the right.

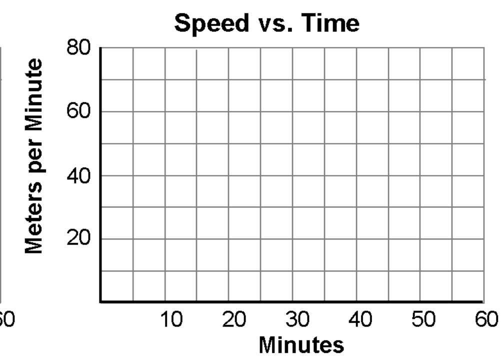
   a. The bicycle trip through hilly country



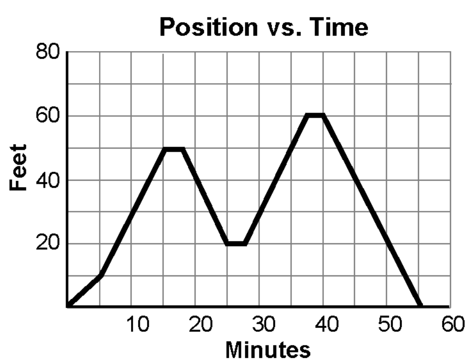


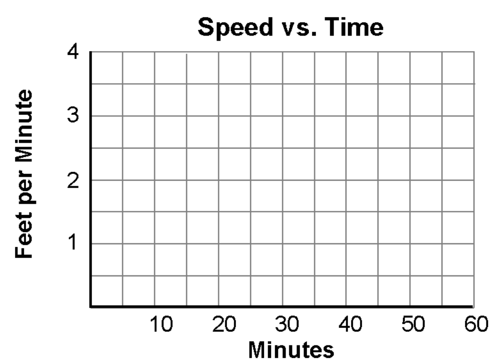
   b. A walk in the park





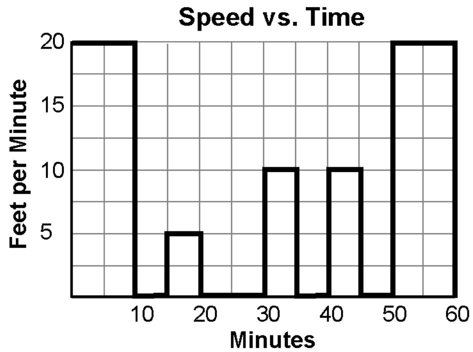
   c. Strolling up and down the supermarket aisles

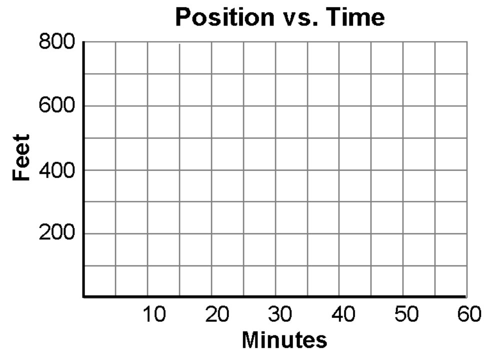




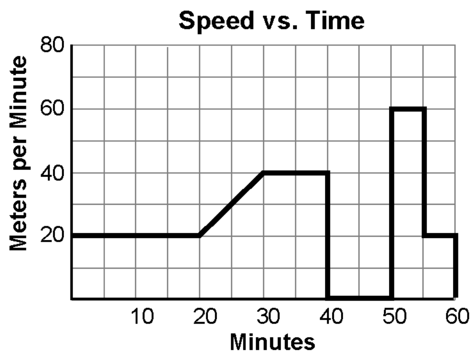
2. For each speed-time graph, calculate and plot the distance on the position-time graph to the right. For this practice, assume that movement is always away from the starting position.

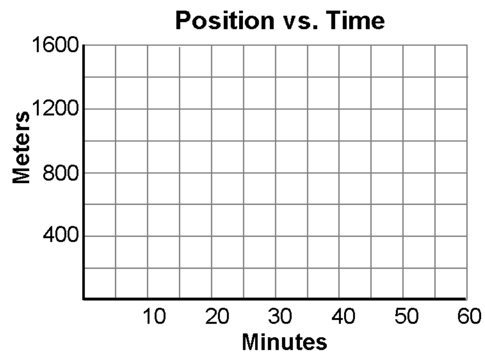
   a. The honey bee among the flowers



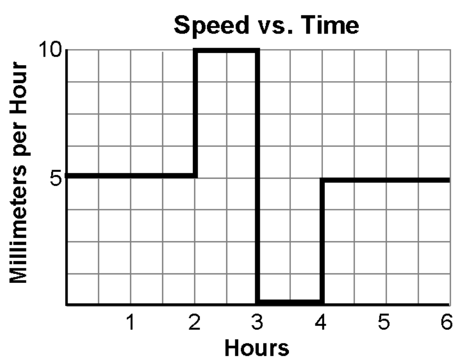


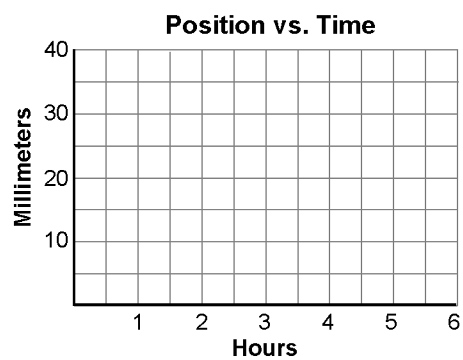
   b. Rover runs the street





   c. The amoeba





Name: ______________________ Date: __________

CONTENT 4.2

4.2 Analyzing Graphs of Motion Without Numbers

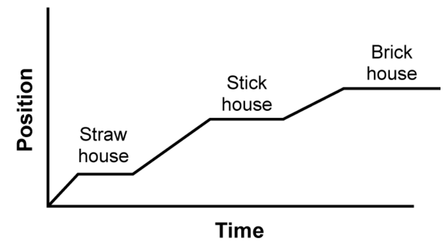
READ

Position-time graphs

The graph at right represents the story of "The Three Little Pigs." The parts of the story are listed below.

- The wolf started from his house. The graph starts at the origin.
- Traveled to the straw house. The line moves upward.
- Stayed to blow it down and eat dinner. The line is flat because position is not changing.
- Traveled to the stick house. The line moves upward again.
- Again stayed, blew it down, and ate seconds. The line is flat.
- Traveled to the brick house. The line moves upward.
- Died in the stew pot at the brick house. The line is flat.

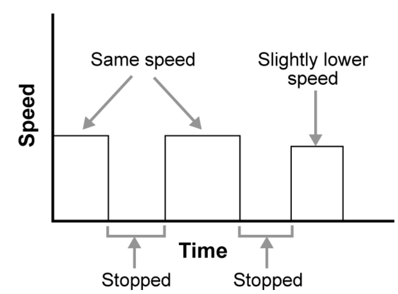
Position-time graph of the wolf in *The Three Little Pigs*



The graph illustrates that the pigs' houses are generally in a line away from the wolf's house and that the brick house was the farthest away.

Speed-time graphs

A speed-time graph displays the speed of an object over time and is based on position-time data. Speed is the relationship between distance (position) and time, $v = d/t$. For the first part of the wolf's trip in the position versus time graph, the line rises steadily. This means the speed for this first leg is constant. If the wolf traveled this first leg faster, the slope of the line would be steeper.



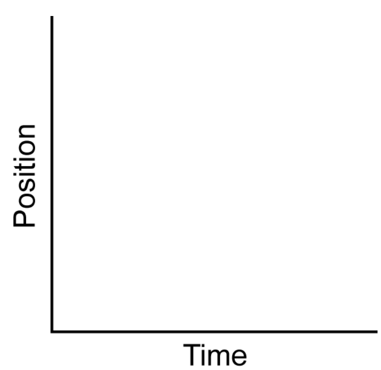
The wolf moved at the same speed toward his first two "visits." His third trip was slightly slower. Except for this slight difference, the wolf was either at one speed or stopped (shown by a flat line in the speed versus time graph).

PRACTICE

Read the steps for each story. Sketch a position-time graph and a speed-time graph for each story.

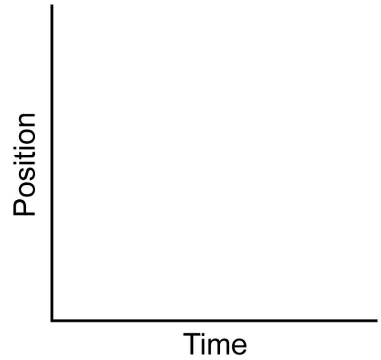
1. Graph Red Riding Hood's movements according to the following events listed in the order they occurred:

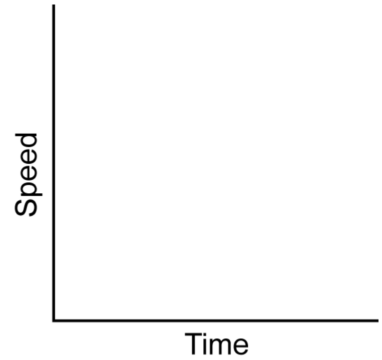
- Little Red Riding Hood set out for Grandmother's cottage at a good walking pace.
- She stopped briefly to talk to the wolf.
- She walked a bit slower because they were talking as they walked to the wild flowers.
- She stopped to pick flowers for quite a while.



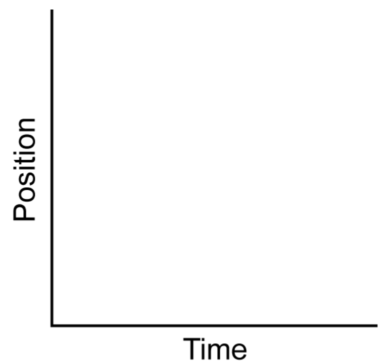
- Realizing she was late, Red Riding Hood ran the rest of the way to Grandmother's cottage.

2. Graph the movements of the Tortoise and the Hare. Use two lines to show the movements of each animal on each graph. The movements of each animal is listed in the order they occurred.
   - The tortoise and the hare began their race from the combined start-finish line. By the end of the race, the two will be at the same position at which they started.
   - Quickly outdistancing the tortoise, the hare ran off at a moderate speed.
   - The tortoise took off at a slow but steady speed.
   - The hare, with an enormous lead, stopped for a short nap.
   - With a startle, the hare awoke and realized that he had been sleeping for a long time.
   - The hare raced off toward the finish at top speed.
   - Before the hare could catch up, the tortoise's steady pace won the race with an hour to spare.

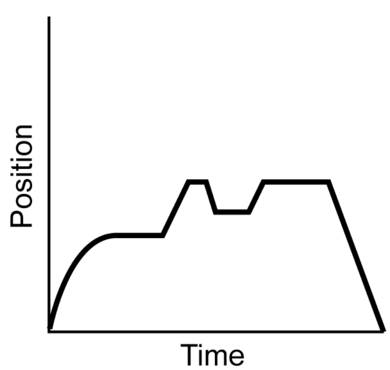


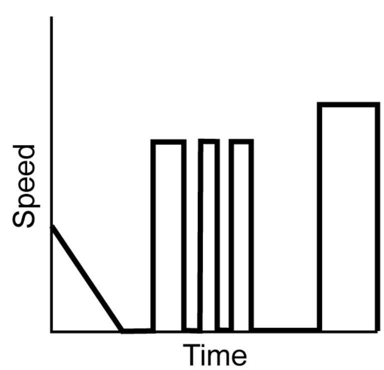


3. Graph the altitude of the sky rocket on its flight according to the following sequence of events listed in order.
   - The sky rocket was placed on the launcher.
   - As the rocket motor burned, the rocket flew faster and faster into the sky.
   - The motor burned out; although the rocket began to slow, it continued to coast even higher.
   - Eventually, the rocket stopped for a split second before it began to fall back to Earth.
   - Gravity pulled the rocket faster and faster toward Earth until a parachute popped out, slowing its descent.
   - The descent ended as the rocket landed gently on the ground.



4. A story told from a graph: Tim, a student at Cumberland School, was determined to ask Caroline for a movie date. Use these graphs of his movements from his house to Caroline's to write the story.





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CONTENT 4.3

4.3 Acceleration

READ

Acceleration is the rate of change in the speed of an object. To determine the rate of acceleration, you use the formula below. The units for acceleration are meters per second per second or m/s^2.

$$\text{Acceleration} = \frac{\text{Final speed} - \text{Beginning speed}}{\text{Time}}$$

$$a = \frac{v_2 - v_1}{t}$$

A positive value for acceleration shows speeding up, and negative value for acceleration shows slowing down. Slowing down is also called *deceleration*.

The acceleration formula can be rearranged to solve for other variables such as final speed (v_2) and time (t).

$$v_2 = v_1 + (a \times t)$$

$$t = \frac{v_2 - v_1}{a}$$

EXAMPLES

1. A skater increases her velocity from 2.0 m/s to 10.0 m/s in 3.0 seconds. What is the skater's acceleration?

<table>
<tr><td>Looking for
Acceleration of the skater</td><td rowspan="3">Solution
$\text{Acceleration} = \frac{10.0\text{ m/s} - 2.0\text{ m/s}}{3.0\text{ s}} = 2.7\text{ m/s}^2$
The acceleration of the skater is 2.7 meters per second per second.</td></tr>
<tr><td>Given
Beginning speed = 2.0 m/s
Final speed = 10.0 m/s
Change in time = 3.0 seconds</td></tr>
<tr><td>Relationship
$a = \frac{v_2 - v_1}{t}$</td></tr>
</table>

2. A car accelerates at a rate of 3.0 m/s^2. If its original speed is 8.0 m/s, how many seconds will it take the car to reach a final speed of 25.0 m/s?

<table>
<tr><td>Looking for
The time to reach the final speed.</td><td rowspan="3">Solution
$\text{Time} = \frac{25.0\text{ m/s} - 8.0\text{ m/s}}{3.0\text{ m/s}^2} = 5.7\text{ s}$
The time for the car to reach its final speed is 5.7 seconds.</td></tr>
<tr><td>Given
Beginning speed = 8.0 m/s; Final speed = 25.0 m/s
Acceleration = 3.0 m/s^2</td></tr>
<tr><td>Relationship
$t = \frac{v_2 - v_1}{a}$</td></tr>
</table>

PRACTICE

1. While traveling along a highway, a driver slows from 24 m/s to 15 m/s in 12 seconds. What is the automobile's acceleration? (Remember that a negative value indicates a slowing down or deceleration.)

2. A parachute on a racing dragster opens and changes the speed of the car from 85 m/s to 45 m/s in a period of 4.5 seconds. What is the acceleration of the dragster?

3. The table below contains data for a ball rolling down a hill. Fill in the missing data values in the table and determine the acceleration of the rolling ball.

Time (seconds)	Speed (km/h)
0 (start)	0 (start)
2	3
	6
	9
8	
10	15

4. A car traveling at a speed of 30.0 m/s encounters an emergency and comes to a complete stop. How much time will it take for the car to stop if it decelerates at $-4.0\ m/s^2$?

5. If a car can go from 0 to 60. mph in 8.0 seconds, what would be its final speed after 5.0 seconds if its starting speed were 50. mph?

6. A cart rolling down an incline for 5.0 seconds has an acceleration of $4.0\ m/s^2$. If the cart has a beginning speed of 2.0 m/s, what is its final speed?

7. A helicopter's speed increases from 25 m/s to 60 m/s in 5 seconds. What is the acceleration of this helicopter?

8. As she climbs a hill, a cyclist slows down from 25 mph to 6 mph in 10 seconds. What is her deceleration?

9. A motorcycle traveling at 25 m/s accelerates at a rate of $7.0\ m/s^2$ for 6.0 seconds. What is the final speed of the motorcycle?

10. A car starting from rest accelerates at a rate of 8.0 m/s. What is its final speed at the end of 4.0 seconds?

11. After traveling for 6.0 seconds, a runner reaches a speed of 10. m/s. What is the runner's acceleration?

12. A cyclist accelerates at a rate of $7.0\ m/s^2$. How long will it take the cyclist to reach a speed of 18 m/s?

13. A skateboarder traveling at 7.0 meters per second rolls to a stop at the top of a ramp in 3.0 seconds. What is the skateboarder's acceleration?

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4.3 Acceleration and Speed-Time Graphs

READ

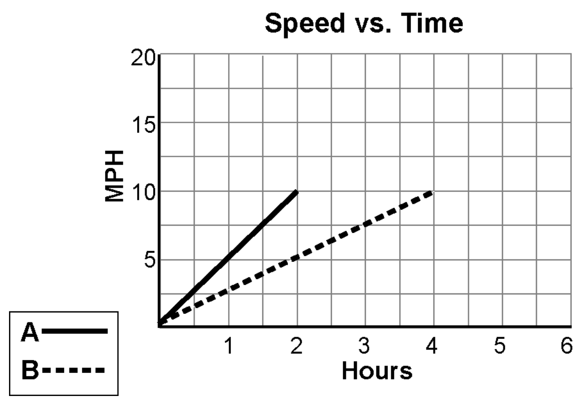
Acceleration is the rate of change in the speed of an object. The graph below shows that object A accelerated from rest to 10 miles per hour in two hours. The graph also shows that object B took four hours to accelerate from rest to the same speed. Therefore, object A accelerated twice as fast as object B.

EXAMPLE

Calculating acceleration from a speed-time graph

The steepness of the line in a speed-time graph is related to acceleration. This angle is the slope of the line and is found by dividing the change in the *y*-axis value by the change in the *x*-axis value.

$$\text{Acceleration} = \frac{\Delta y}{\Delta x}$$



In everyday terms, we can say that the speed of object A "increased 10 miles per hour in two hours." Using the slope formula:

$$\text{Acceleration} = \frac{\Delta y}{\Delta x} = \frac{10\text{ mph} - 0\text{ mph}}{2\text{ hours} - 0\text{ hour}} = \frac{5\text{ mph}}{\text{hour}}$$

- Acceleration = $\Delta y/\Delta x$ (the symbol Δ means "change in")
- Acceleration = (10 mph – 0 mph)/(2 hours – 0 hours)
- Acceleration = 5 mph/hour (read as 5 miles per hour per hour)

The double *per time* label attached to all accelerations may seem confusing at first. It is not so alien a concept if you break it down into its parts:

The speed changes. . .	. . . during this amount of time:
5 miles per hour	each hour

Accelerations can be negative. If the line slopes downward, Δy will be a negative number because a larger value of y will be subtracted from a smaller value of y.

Calculating distance from a speed-time graph

The area between the line on a speed-time graph and the baseline is equal to the distance that an object travels. This follows from the rate formula:

$$\text{Rate or Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$v = \frac{d}{t}$$

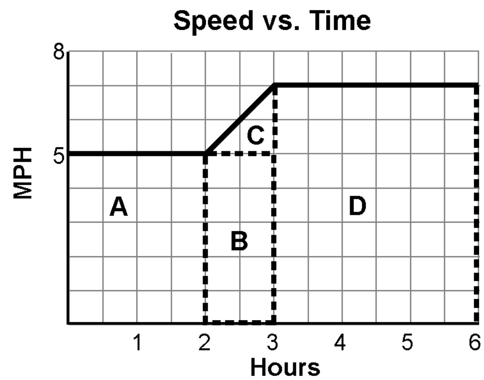
Or, rewritten:

$$vt = d$$

$$\text{miles}/\cancel{\text{hour}} \times 3\ \cancel{\text{hours}} = 3\ \text{miles}$$

Notice how the labels cancel to produce a new label that fits the result.

Here is a speed-time graph of a boat starting from one place and sailing to another:



The graph shows that the sailboat accelerated between the second and third hour. We can find the total distance by finding the area between the line and the baseline. The easiest way to do that is to break the area into sections that are easy to solve and then add them together.

$$A + B + C + D = \text{distance}$$

- Use the formula for the area of a rectangle, $A = L \times W$, to find areas A, B, and D.
- Use the formula for finding the area of a triangle, $A = l \times w/2$, to find area C.

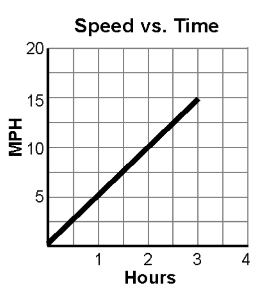
$$A + B + C + D = \text{distance}$$

$$10\ \text{miles} + 5\ \text{miles} + 1\ \text{mile} + 21\ \text{miles} = 37\ \text{miles}$$

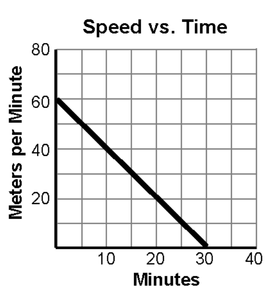
PRACTICE

Calculate acceleration from each of these graphs.

1. Graph 1:

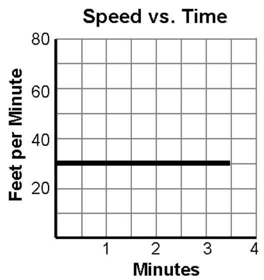


2. Graph 2:

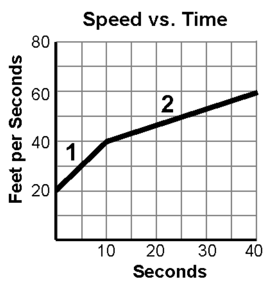


CONTENT 4.3

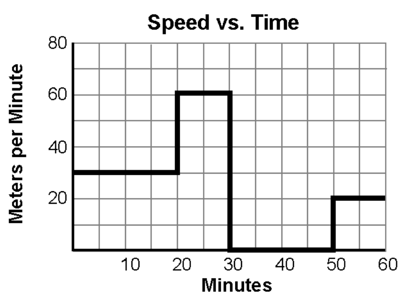
3. Graph 3:



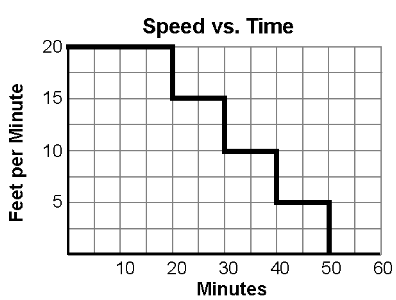
4. Find acceleration for segment 1 and segment 2 in this graph:



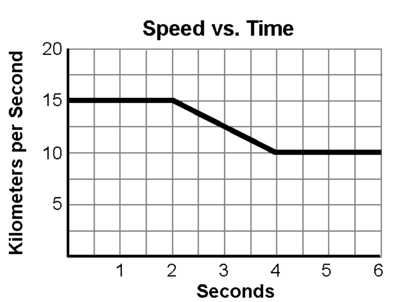
5. Calculate total distance for this graph:



6. Calculate total distance for this graph:



7. Calculate total distance for this graph:



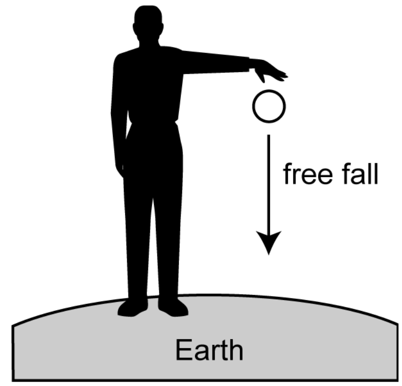
Name: ____________________ Date: __________

CHALLENGE! 4.3

4.3 Acceleration Due to Gravity

READ

Acceleration due to gravity is known to be 9.8 meters/second/second or 9.8 m/s^2 and is represented by *g*. Three conditions must be met before we can use this value:
(1) the object must be in free fall
(2) the object must have negligible air resistance, and
(3) the object must be close to the surface of Earth.



In all of the examples and problems, we will assume that these conditions have been met. Remember that speed refers to "how fast" in any direction, but velocity refers to "how fast" in a specific direction. The sign of numbers in these calculations is important. Velocities upward shall be positive, and velocities downward shall be negative. Because the *y*-axis of a graph is vertical, change in height shall be indicated by *y*.

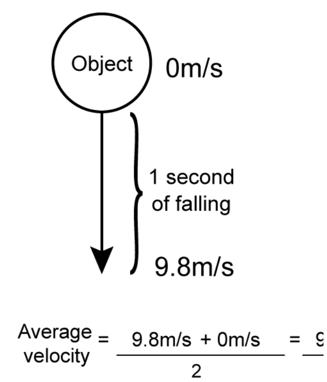
Here is the equation for solving for velocity:

$$\text{final velocity} = \text{initial velocity} + (\text{the acceleration due to the force of gravity} \times \text{time})$$

OR

$$v = v_0 + gt$$

Imagine that an object falls for one second. We know that at the end of the second it will be traveling at 9.8 meters/second. However, it began its fall at zero meters/second. Therefore, its average velocity is half of 9.8 meters/second. We can find distance by multiplying this average velocity by time.



Here is the equation for solving for distance. See if you can find these concepts in the equation:

$$\text{distance} = \frac{\text{the acceleration due to the force of gravity} \times \text{time}}{2} \times \text{time}$$

OR

$$y = \frac{1}{2}gt^2$$

EXAMPLE

Example 1: How fast will a pebble be traveling 3.0 seconds after being dropped?

$$v = v_0 + gt$$

$$v = 0 + (-9.8 \text{ m/s}^2 \times 3.0 \text{ s})$$

$$v = -29 \text{ m/s}$$

(Note that *gt* is negative because the direction is downward.)

Example 2: A pebble dropped from a bridge strikes the water in 4.0 seconds. How high is the bridge?

$$y = \frac{1}{2}gt^2$$

$$y = \frac{1}{2} \times 9.8 \text{ m/s} \times 4.0 \text{ s} \times 4.0 \text{ s}$$

$$y = \frac{1}{2} \times 9.8 \text{ m/s}^2 \times 4.0 \text{ s} \times 4.0 \text{ s}$$

$$y = 78.4 \text{ meters}$$

Note that the seconds cancel. The answer written with the correct number of significant figures is 78 meters. The bridge is 78 meters high.

PRACTICE

1. A penny dropped into a wishing well reaches the bottom in 1.50 seconds. What was the velocity at impact?
2. A pitcher threw a baseball straight up at 35.8 meters per second. What was the ball's velocity after 2.50 seconds? (Note that, although the baseball is still climbing, gravity is accelerating it downward.)
3. In a bizarre but harmless accident, a watermelon fell from the top of the Eiffel Tower. How fast was the watermelon traveling when it hit the ground 7.80 seconds after falling?
4. A water balloon was dropped from a high window and struck its target 1.1 seconds later. If the balloon left the person's hand at –5.0 meters per second, what was its velocity on impact?
5. A stone tumbles into a mine shaft and strikes bottom after falling for 4.2 seconds. How deep is the mine shaft?
6. A boy threw a small bundle toward his girlfriend on a balcony 10. meters above him. The bundle stopped rising in 1.5 seconds. How high did the bundle travel? Was that high enough for her to catch it?

The equations demonstrated so far can be used to find time of flight from speed or distance, respectively. Remember that an object thrown into the air represents two mirror-image flights, one up and the other down.

	Original equation	Rearranged equation to solve for time
Time from velocity	$v = v_0 + gt$	$t = \frac{v - v_0}{g}$
Time from distance	$y = \frac{1}{2}gt^2$	$t = \sqrt{\frac{2y}{g}}$

7. At about 55 meters per second, a falling parachuter (before the parachute opens) no longer accelerates. Air friction opposes acceleration. Although the effect of air friction begins gradually, imagine that the parachuter is free falling until terminal speed (the constant falling speed) is reached. How long would that take?

8. The climber dropped her compass at the end of her 240-meter climb. How long did it take to strike bottom?

Name: ______________________________ Date: __________

MATH 5.1

5.1 Ratios and Proportions

READ

Professional chefs use ratios and proportions daily to figure out how much of various ingredients they will need to make a particular dish. They may serve the same dessert one day to a wedding party with 300 guests, and another day to a dinner party for eight people. Ratios and proportions help them figure out the right quantities of ingredients to buy for each meal. In this skill sheet, you will practice converting a recipe for different group sizes.

A recipe for Double Fudge Brownies

Ingredients:

$^3/_4$ c. sugar	2 eggs
6 tablespoons unsalted butter	1 teaspoon vanilla extract
2 tablespoons milk	$^3/_4$ cup all-purpose flour
2 cups semi-sweet chocolate chips	$^1/_3$ teaspoon baking soda
$^1/_4$ teaspoon salt	2 tablespoons confectioner's sugar

Makes 16 brownies.

EXAMPLES

1. What is the ratio of milk to chocolate chips in the recipe above? $\frac{\text{2 tablespoons}}{\text{2 cups}}$

2. When we know the ratios, we can make proportions by setting two ratios equal to one another. This will help us to find missing answers.

   Suppose Patricia needs only 8 brownies and doesn't want any leftovers. Find out how much of each ingredient she needs. The original recipe will make 16 brownies. You will use the ratio of $^8/_{16} = {^1/_2}$ to find the amount for each of the ingredients. Use cross-multiplication to solve the proportions.

For flour:

Step 1 $\frac{8}{16} = \frac{x}{3/4}$

Step 2 $8 \times \frac{3}{4} = 16x$

Step 3 $6 = 16x$

Step 4 $\frac{6}{16} = \frac{16x}{16}$

Step 5 $\frac{3}{8} = x$

Patricia needs $^3/_8$ cup of flour to make 8 brownies.

MATH 5.1

PRACTICE

1. What is the ratio of unsalted butter to eggs?

   For every __________ tablespoons of butter, you will need __________ eggs.

2. What is the ratio of flour to baking soda?

   For every __________ cups of flour, you will need __________ teaspoon of baking soda.

3. What is the ratio of salt to flour?
   For every __________ teaspoon(s) of salt, you will need __________ cups of flour

4. Find the correct amount of each ingredient to make 8 brownies ($^1/_2$ of the recipe).

Flour	$^3/_8$ cup
Sugar	
Butter	
Milk	
Chocolate chips	
Eggs	
Vanilla extract	
Baking soda	
Salt	
Confectioner's sugar	

5. Why are the eggs and confectioner's sugar amounts easy to work with to make 8 brownies?

6. Patricia has a little extra of all the ingredients. How many brownies can be made using 3 cups of chocolate chips?

7. How much vanilla will she need when she makes the batch of brownies using 3 cups of chocolate chips?

Name: ________________________ Date: __________

READING 5.1

5.1 Internet Research Skills

READ

The Internet is a valuable tool for finding answers to your questions about the world. A search engine is like an on-line index to information on the World Wide Web. There are many different search engines to choose from. Search engines differ in how often they are updated, how many documents they contain in their index, and how they search for information. Your teacher may suggest several search engines for you to try.

EXAMPLE

Search engines ask you to type a word or phrase into a box known as a *field*. Knowing how search engines work can help you pinpoint the information you need. However, if your phrase is too vague, you may end up with a lot of unhelpful information.

How could you find out who was the first woman to participate in a space shuttle flight?

First, put **key phrases** in quotation marks. You want to know about the "first woman" on a "space shuttle." Quotation marks tell the engine to search for those words together.

Second, if you only want websites that contain both phrases, **use a + sign** between them. Typing **"first woman" + "space shuttle"** into a search engine will limit your search to websites that contain both phrases.

If you want to broaden your search, use the word **or** between two terms. For example, if you type **"first female" or "first woman" + "space shuttle"** the search engine will list any website that contains either of the first two phrases, as long as it also contains the phrase "space shuttle."

You can narrow a search by using the word **not.** For example, if you wanted to know about marine mammals other than whales, you could type **"marine mammals" not "whales"** into the field. Please note that some search engines use the minus sign (-) rather than the word **not**.

PRACTICE

1. If you wanted to find out about science museums in your state that are not in your own city or town, what would you type into the search engine?
2. If you wanted to find out which dog breeds are inexpensive, what would you type into the search engine?
3. How could you research alternatives to producing electricity through the combustion of coal or natural gas?

READ

The quality of information found on the Internet varies widely. This section will give you some things to think about as you decide which sources to use in your research.

1. **Authority:** How well does the author know the subject matter? If you search for "Newton's laws" on the Internet, you may find a science report written by a fifth grade student, and a study guide written by a college professor. Which website is the most authoritative source?
   Museums, national libraries, government sites, and major, well-known "encyclopedia sources" are good places to look for authoritative information.
2. **Bias:** Think about the author's purpose. Is it to inform, or to persuade? Is it to get you to buy something? Comparing several authoritative sources will help you get a more complete understanding of your subject.
3. **Target audience:** For whom was this website written? Avoid using sites designed for students well below your grade level. You need to have an understanding of your subject matter at or above your own grade level. Even authoritative sites for younger students (children's encyclopedias, for example) may leave out details and simplify concepts in ways that would leave gaps in your understanding of your subject.
4. **Is the site up-to-date, clear, and easy to use?** Try to find out when the website was created, and when it was last updated. If the site contains links to other sites, but those links don't work, you may have found a site that is infrequently or no longer maintained. It may not contain the most current information about your subject. Is the site cluttered with distracting advertisements? You may wish to look elsewhere for the information you need.

PRACTICE

1. What is your favorite sport or activity? Search for information about this sport or activity. List two sites that are authoritative and two sites that are not authoritative. Explain your reasoning. Finally, write down the best site for finding out information about your favorite sport.
2. Search for information about a physical science topic of your choice on the Internet (*i.e.*, "simple machines," "Newton's Laws," "Galileo"). Find one source that you would NOT consider authoritative. Write the key words you used in your search, the web address of the source, and a sentence explaining why this source is not authoritative.
3. Find a different source that is authoritative, but intended for a much younger audience. Write the web address and a sentence describing who you think the intended audience is.
4. Find three sources that you would consider to be good choices for your research here. Write two to three sentence description of each. Describe the author, the intended audience, the purpose of the site, and any special features not found in other sites.

Name: ______________________________ Date: __________

READING 5.1

5.1 Preparing a Bibliography

READ

When you write a research paper or prepare a presentation for your class, it is important to document your sources. A bibliography serves two purposes. First, a bibliography gives credit to the authors who wrote the material you used to learn about your subject. Second, a bibliography provides your audience with sources they can use if they would like to learn more about your subject.

This skill sheet provides bibliography formats and examples for various types of research materials you may use when preparing science papers and presentations.

EXAMPLES

Books:

Author last name, First name. (Year published). *Title of book*. Place of publication: Name of publisher.

Vermeij, Geerat. (1997). *Privileged Hands: A Scientific Life*. New York: W.H. Freeman and Company.

Newspaper and Magazine Articles:

Author listed:

Author last name, First name. (Date of publication). Title of Article. *Title of Newspaper or Magazine*, page # or #'s.

Searcy, Dionne. (2006, March 20). Wireless Internet TV Is Launched in Oklahoma. *The Wall Street Journal*, p. B4.

Brody, Jane. (2006, February/March). 10 Kids' Nutrition Myth Busters. *Nick Jr Family Magazine*, pp. 72–73.

No author listed:

Title of article. (Date of publication). *Title of Newspaper or Magazine*, page # or #'s.

Chew on this: Gum may speed recovery. (2006, March 20). *St. Louis Post-Dispatch*, p.H2.

Adventures in Turning Trash into Treasure: (2006, April). *Reader's Digest*, p. 24.

Online Newspaper or Magazine:

Author listed:

Author last name, First name. (Date of publication). Title of Article. *Title of Newspaper or Magazine*, Retrieved date, from web address.

Dybas, Cheryl Lyn. (2006, March 20). Early Spring Disturbing Life on Northern Rivers. *The Washington Post*, Retrieved March 22, 2006, from www.washingtonpost.com.

No author listed:

Title of Article. (Date of publication). *Title of Newspaper or Magazine*. Retrieved date, from web address.

Comet mystery turns from hot to cold. (2006, March 20). *The Boston Globe*, retrieved March 22, 2006, from Boston.com.

Online document:

Author listed:

Author last name, author first name. (Date of publication). *Title of document*. Retrieved date, from web address.

Martinez, Carolina. (2006, March 9). *NASA's Cassini Discovers Potential Liquid Water on Enceladus*. Retrieved March 22, 2006, from http://www.nasa.gov/mission_pages/cassini/media/cassini-20060309.html

No Author listed:

Organization responsible for website. (Date of publication). *Title of document*. Retrieved date, from web address.

National Science Foundation. (2005, December 15). *A fish of a different color*. Retrieved March 22, 2206 from http://www.nsf.gov/news/news_summ.jsp?cntn_id=105661&org=NSF&from=news.

Name: ______________________ Date: __________

CONTENT 5.1

5.1 Mass, Weight, and Gravity

READ

How do we define mass, weight, and gravity?

mass	weight	gravity
• Mass is a measure of the amount of matter in an object. Mass is not related to gravity. • The mass of an object does not change when it is moved from one place to another. • Mass is commonly measured in grams or kilograms.	• Weight is a measure of the gravitational force between two objects. • The weight of an object does change when the amount of gravitational force changes, as when an object is moved from Earth to the moon. • Weight is commonly measured in newtons or pounds.	• The force that causes all masses to attract one another. The strength of the force depends on the size of the masses and their distance apart.

How are mass, weight, and gravity related?

The weight equation $W = mg$ shows that an object's weight (in newtons) is equal to its mass (in kilograms) multiplied by the strength of gravity (in newtons per kilogram) where the object is located. The weight equation can be rearranged to find weight, mass, or the strength of gravity if you know any two of the three.

Use...	if you want to find...	and you know...
$W = mg$	weight (W)	mass (m) and strength of gravity (g)
$m = W/g$	mass (m)	weight (W) and strength of gravity (g)
$g = W/m$	strength of gravity (g)	weight (W) and mass (m)

EXAMPLE

- Calculate the weight (in newtons) of a 5.0-kilogram backpack on Earth (g = 9.8 N/kilogram).

  Solution:

$$W = m(g)$$
$$W = (5.0\text{ kg})(9.8\text{ N/kg})$$
$$W = 49\text{ N}$$

- The same backpack weighs 8.2 newtons on Earth's moon. What is the strength of gravity on the moon?

  Solution:

$$g = W/m$$
$$g = 8.2\text{ N} / 5.0\text{ kg}$$
$$g = 1.6\text{ N/kg}$$

PRACTICE

1. A physical science textbook has a mass of 2.2 kilograms.
   a. What is its weight on Earth?
   b. What is its weight on Mars? (g = 3.7 N/kg)
   c. If the textbook weighs 19.6 newtons on Venus, what is the strength of gravity on that planet?
2. An astronaut weighs 104 newtons on the moon, where the strength of gravity is 1.6 newtons per kilogram.
   a. What is her mass?
   b. What is her weight on Earth?
   c. What would she weigh on Mars?
3. Of all the planets in our solar system, Jupiter has the greatest gravitational strength.
   a. If a 0.500-kilogram pair of running shoes would weigh 11.55 newtons on Jupiter, what is the strength of gravity there?
   b. If the same pair of shoes weighs 0.3 newtons on Pluto (a dwarf planet), what is the strength of gravity there?
   c. What does the pair of shoes weigh on Earth?
4. A tractor-trailer truck carrying boxes of toy rubber ducks stops at a weigh station on the highway. The driver is told that the truck weighs 44,000. pounds.
   a. If there are 4.448 newtons in a pound, what is the weight of the toy-filled truck in newtons?
   b. What is the mass of the toy-filled truck?
   c. The truck drops off its load of toys, then stops at a second weigh station. Now the truck weighs 33,000. pounds. What is its weight in newtons?
   d. Challenge! Find the total mass of the rubber duck-filled boxes that were carried by the truck.

Name: ______________________________ Date: __________

CHALLENGE! 5.1

5.1 Gravity Problems

READ

In this skill sheet, you will practice using proportions as you learn more about the strength of gravity on different planets.

PRACTICE

Comparing the strength of gravity on the planets

Table 1 lists the strength of gravity on each planet in our solar system. We can see more clearly how these values compare to each other using proportions. First, we assume that Earth's gravitational strength is equal to "1." Next, we set up the proportion where x equals the strength of gravity on another planet (in this case, Mercury) as compared to Earth.

$$\frac{1}{\text{Earth gravitational strength}} = \frac{x}{\text{Mercury gravitational strength}}$$

$$\frac{1}{9.8\ \text{N/kg}} = \frac{x}{3.7\ \text{N/kg}}$$

$$(1 \times 3.7\ \text{N/kg}) = (9.8\ \text{N/kg} \times x)$$

$$\frac{3.7\ \text{N/kg}}{9.8\ \text{N/kg}} = x$$

$$0.38 = x$$

Note that the units cancel. The result tells us that Mercury's gravitational strength is a little more than a third of Earth's. Or, we could say that Mercury's gravitational strength is 38% as strong as Earth's.

Now, calculate the proportions for the remaining planets.

Table 1: The strength of gravity on planets in our solar system

Planet	Strength of gravity (N/kg)	Value compared to Earth's gravitational strength
Mercury	3.7	0.38
Venus	8.9	
Earth	9.8	1
Mars	3.7	
Jupiter	23.1	
Saturn	9.0	
Uranus	8.7	
Neptune	11.0	
Pluto	0.6	

How much does it weigh on another planet?

Use your completed Table 1 to solve the following problems.

Example:

- A bowling ball weighs 15 pounds on Earth. How much would this bowling ball weigh on Mercury?

$$\frac{\text{Weight on Earth}}{\text{Weight on Mercury}} = \frac{1}{0.38}$$

$$\frac{1}{0.38} = \frac{15\text{ pounds}}{x}$$

$$0.38 \times 15\text{ pounds} = x$$

$$x = 5.7\text{ pounds}$$

1. A cat weighs 8.5 pounds on Earth. How much would this cat weigh on Neptune?

2. A baby elephant weighs 250 pounds on Earth. How much would this elephant weigh on Saturn? Give your answer in newtons (4.48 newtons = 1 pound).

3. On Pluto, a baby would weigh 2.7 newtons. How much does this baby weigh on Earth? Give your answer in newtons and pounds.

4. Imagine that it is possible to travel to each planet in our solar system. After a space "cruise," a tourist returns to Earth. One of the ways he recorded his travels was to weigh himself on each planet he visited. Use the list of these weights on each planet to figure out the order of the planets he visited. On Earth he weighs 720 newtons. List of weights: 655 N; 1,872 N; 792 N; 36 N; and 661 N.

Challenge: Using the Universal Law of Gravitation

Here is an example problem that is solved using the equation for Universal Gravitation.

Equation of Universal Gravitation:

$$F = G\frac{m_1 m_2}{R^2}$$

Force (N)

Mass 1 (kg)

Mass 2 (kg)

Gravitational constant (6.67×10^{11} N•m²/kg²)

Distance between mass 1 and mass 2 (m)

Example

What is the force of gravity between Pluto and Earth? The mass of Earth is 6.0×10^{24} kg. The mass of Pluto is 1.3×10^{22} kg. The distance between these two planets is 5.76×10^{12} meters.

$$\text{Force of gravity between Earth and Pluto} = \left(\frac{6.67 \times 10^{-11}\text{N-m}^2}{\text{kg}^2}\right)\frac{(6.0 \times 10^{24}\text{kg}) \times (1.3 \times 10^{22}\text{kg})}{(5.76 \times 10^{12}\text{m})^2}$$

$$\text{Force of gravity} = \frac{52.0 \times 10^{35}}{33.2 \times 10^{24}} = 1.57 \times 10^{11}\text{N}$$

Now use the equation for Universal Gravitation to solve this problem:

5. What is the force of gravity between Jupiter and Saturn? The mass of Jupiter is 6.4×10^{24} kg. The mass of Saturn is 5.7×10^{26} kg. The distance between Jupiter and Saturn is 6.52×10^{11} m.

Name: ______________________ Date: __________

CHALLENGE!
5.1

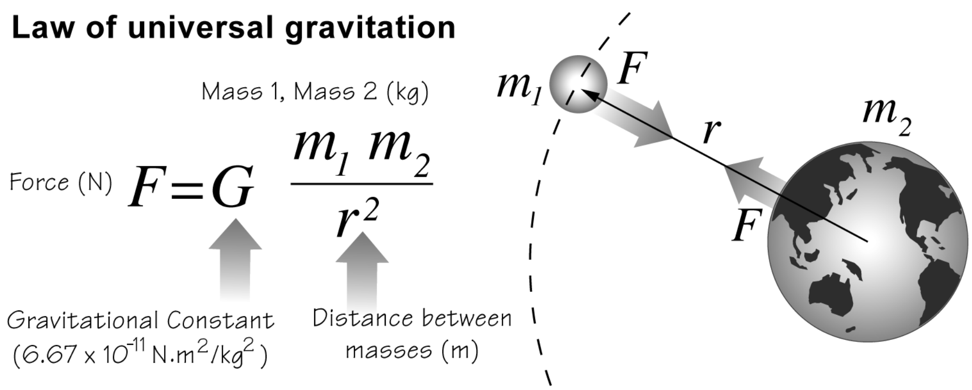
5.1 Universal Gravitation

READ

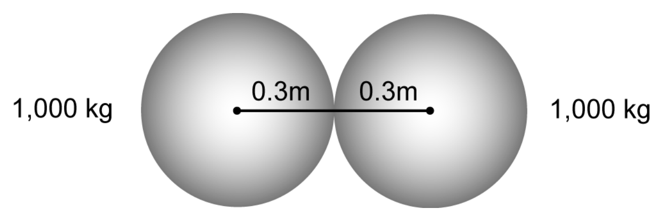
The law of universal gravitation allows you to calculate the gravitational force between two objects from their masses and the distance between them. The law includes a value called the gravitational constant, or "G." This value is the same everywhere in the universe. Calculating the force between small objects like grapefruits or huge objects like planets, moons, and stars is possible using this law.

What is the law of universal gravitation?

The force between two masses m_1 and m_2 that are separated by a distance r is given by:



So, when the masses m_1 and m_2 are given in kilograms and the distance r is given in meters, the force has the unit of newtons. Remember that the distance r corresponds to the distance between the center of gravity of the two objects.



For example, the gravitational force between two spheres that are touching each other, each with a radius of 0.300 meter and a mass of 1,000. kilograms, is given by:

$$F = 6.67 \times 10^{-11}\text{N-m}^2/\text{kg}^2 \; \frac{1{,}000.\text{ kg} \times 1{,}000.\text{ kg}}{(0.300\text{ m} + 0.300\text{ m})^2} = 0.000185\text{ N}$$

Note: A small car has a mass of approximately 1,000. kilograms. Try to visualize this much mass compressed into a sphere with a diameter of 0.300 meters (30.0 centimeters). If two such spheres were touching one another, the gravitational force between them would be only 0.000185 newtons. On Earth, this corresponds to the weight of a mass equal to only 18.9 milligrams. The gravitational force is not very strong!

PRACTICE

Answer the following problems. Write your answers using scientific notation.

1. Calculate the force between two objects that have masses of 70. kilograms and 2,000. kilograms. Their centers of gravity are separated by a distance of 1.00 meter.

2. Calculate the force between two touching grapefruits each with a radius of 0.080 meters and a mass of 0.45 kilograms.

3. Calculate the force between one grapefruit as described above and Earth. Earth has a mass of 5.9742×10^{24} kilograms and a radius of 6.3710×10^6 meters. Assume the grapefruit is resting on Earth's surface.

4. A man on the moon with a mass of 90. kilograms weighs 146 newtons. The radius of the moon is 1.74×10^6 meters. Find the mass of the moon.

5. For $m = 5.9742 \times 10^{24}$ kilograms and $r = 6.3710 \times 10^6$ meters, what is the value given by: $G\frac{m}{r^2}$?
   a. Write down your answer and simplify the units.
   b. What does this number remind you of?
   c. What real-life values do *m* and *r* correspond to?

6. The distance between the centers of Earth and its moon is 3.84×10^8 meters. Earth's mass is 5.9742×10^{24} kilograms and the mass of the moon is 7.36×10^{22} kilograms. What is the force between Earth and the moon?

7. A satellite is orbiting Earth at a distance of 35.0 kilometers. The satellite has a mass of 500. kilograms. What is the force between the planet and the satellite?

8. The mass of the sun is 1.99×10^{30} kilograms and its distance from Earth is 150. million kilometers ($150. \times 10^9$ meters). What is the gravitational force between the sun and Earth?

Name: ______________________________ Date: __________

CONTENT 5.2

5.2 Friction

READ

Just about every move we make involves friction of some sort. This skill sheet will provide you with the opportunity to practice identifying the friction force(s) involved in real-world situations.

EXAMPLE

Marco and his dad are unloading cinder blocks from the back of their pickup truck. They need to haul the blocks across the grass to their backyard, where they are going to make a sandbox for Marco's younger sister. Marco would like to haul a bunch of blocks at once. In the garage, he finds a plastic sled and his sister's red wagon.

- Which type of friction would resist Marco's motion if he pulled the blocks in the sled?

  Solution: Sliding friction.

PRACTICE

1. Answer these additional questions about Marco's sandbox building project.
   a. Which type of friction would resist Marco's motion if he pulled the blocks in the wagon?
   b. Do you think it would take more force to transport five blocks in the sled or in the wagon? Why?
   c. Would the friction force increase, decrease, or stay the same if Marco added two more blocks to the sled or wagon? Explain your answer.
   d. Marco tries piling twelve cinder blocks into the wagon. He pulls and pulls but the wagon doesn't move. What type of force is resisting motion now?
2. Brianna is rowing a small boat across a pond. The air is calm; there is no wind blowing.
   a. What type of friction is resisting her motion?
   b. If two friends join her in the boat, will the friction force change? Why or why not?
3. A freight train speeds along the railroad tracks at 150 km/hr.
   a. Name two types of friction resisting this motion.
   b. If this train were replaced with a mag-lev train, which type of friction would be eliminated?
4. **Research:** Some sports cars are designed with rear spoiler to make the car more stable when turning, accelerating, and braking.
   a. Use the Internet or your local library to find an illustration of a spoiler to share with your class.
   b. Does the spoiler increase or decrease friction between the rear tires and the road?
   c. Some small hybrid cars and sport utility vehicles also have spoilers. What is their purpose? Is it the same or different from the spoiler on a sports car?

Name: ____________________ Date: __________

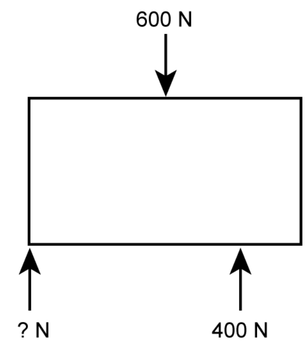
CONTENT 5.3

5.3 Equilibrium

READ

When all forces acting on a body are balanced, the forces are in equilibrium. This skill sheet provides free-body diagrams for you to use for practice in working with equilibrium.

Remember that an unbalanced force results in acceleration. Therefore, the forces acting on an object that is not accelerating must be balanced. These objects may be at rest, or they could be moving at a constant velocity. Either way, we say that the forces acting on these objects are in *equilibrium*.



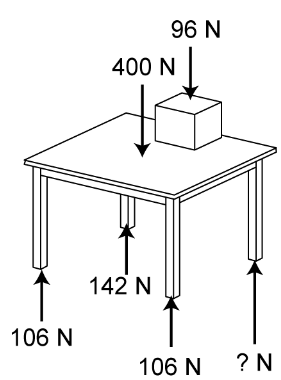
EXAMPLE

What force is necessary in the free-body diagram at right to achieve equilibrium?

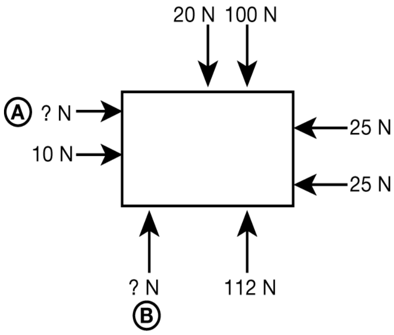
Looking for The unknown force: **? N**	**Solution** 600 N = 400 N + **? N** 600 N – 400 N = 400 N – 400 N + **? N** 200 N = **? N**
Given 600 N is pressing down on the box. 400 N is pressing up on the box.	
Relationship You can solve equilibrium problems using simple equations: 600 N = 400 N + **? N**	

PRACTICE

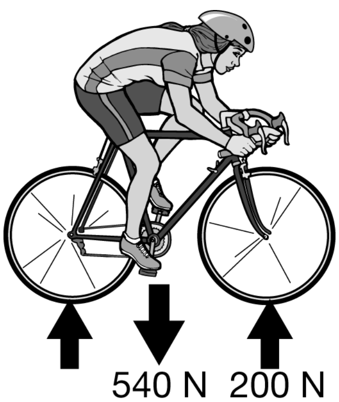
1. Supply the missing force necessary to achieve equilibrium.



2. Supply the missing forces necessary to achieve equilibrium.

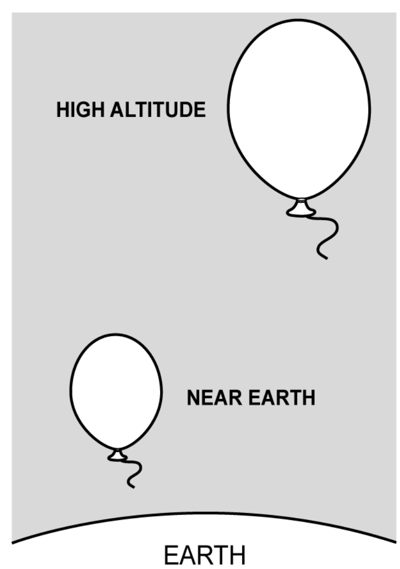


3. In the picture, a girl with a weight of 540 N is balancing on her bike in equilibrium, not moving at all. If the force exerted by the ground on her front wheel is 200 N, how much force is exerted by the ground on her back wheel?



Challenge Question:

4. Helium balloons stay the same size as you hold them, but swell and burst as they rise to high altitudes when you let them go. Draw and label force arrows inside and/or outside the balloons on the graphic at right to show why the near Earth balloon does not burst, but the high altitude balloon does eventually burst. Hint: What are the forces on the inside of the balloon? What are the forces on the outside of the balloons?



Name: ____________________________ Date: __________

CONTENT 6.1

6.1 Net Force and Newton's First Law

READ

Newton's first law tells us that when the net force is zero, objects at rest stay at rest and objects in motion keep moving with the same speed and direction. Changes in motion come from unbalanced forces.

In this skill sheet, you will practice identifying balanced and unbalanced forces in everyday situations.

EXAMPLE

- An empty shopping cart is pushed along a grocery store aisle at constant velocity. Find the cart's weight and the friction force if the shopper produces a force of 40.0 newtons between the wheels and the floor, and the normal force on the cart is 105 newtons.

1. Looking for:	You are asked for the cart's weight and the friction force.
2. Given:	You are given the normal force and the force produced by the shopper pushing the cart.
3. Relationships:	Newton's first law states that if the shopping cart is moving at a constant velocity, the net force must be zero.
4. Solution:	The weight of the cart balances the normal force. Therefore, the weight of the cart is a downward force: -105 N. The forward force produced by the shopper balances the friction force, so the friction force is -40.0 N.

PRACTICE

1. Identify the forces on the same cart at rest.
2. While the cart is moving along an aisle, it comes in contact with a smear of margarine that had recently been dropped on the floor. Suddenly the friction force is reduced from -40.0 newtons to -20.0 newtons. What is the net force on the cart if the "pushing force" remains at 40.0 newtons? Does the grocery cart move at constant velocity over the spilled margarine?
3. Identify the normal force on the shopping cart after 75 newtons of groceries are added to the cart.
4. The shopper pays for his groceries and pushes the shopping cart out of the store, where he encounters a ramp that helps him move the cart from the sidewalk down to the parking lot. What force accelerates the cart down the ramp?
5. Compare the friction force on the cart when it is rolling along the blacktop parking lot to the friction force on the cart when it is inside the grocery store (assume the flooring is smooth vinyl tile).
6. Why is it easy to get one empty cart moving but difficult to get a line of 20 empty carts moving?

Name: ____________________ Date: __________

BIOGRAPHY 6.1

6.1 Isaac Newton

Isaac Newton is one of the most brilliant figures in scientific history. His three laws of motion are probably the most important natural laws in all of science. He also made vital contributions to the fields of optics, calculus, and astronomy.

Plague provides opportunity for genius

Isaac Newton was born in 1642 in Lincolnshire, England. His childhood years were difficult. His father died just before he was born. When he was three, his mother remarried and left her son to live with his grandparents. Newton bitterly resented his stepfather throughout his life.

An uncle helped Newton remain in school and in 1661, he entered Trinity College at Cambridge University. He earned his bachelor's degree in 1665.

Ironically, it was the closing of the university due to the bubonic plague in 1665 that helped develop Newton's genius. He returned to Lincolnshire and spent the next two years in solitary academic pursuit.

During this period, he made significant advances in calculus, worked on a revolutionary theory of the nature of light and color, developed early versions of his three laws of motion, and gained new insights into the nature of planetary motion.

Fear of criticism stifles scientist

When Cambridge reopened in 1667, Newton was given a minor position at Trinity and began his academic career. His studies in optics led to his invention of the reflecting telescope in the early 1670s. In 1672, his first public paper was presented, on the nature of light and color.

Newton longed for public recognition of his work but dreaded criticism. When another bright young scientist, Robert Hooke, challenged some of his points, Newton was furious. An angry exchange of words left Newton reluctant to make public more of his work.

Revolutionary law of universal gravitation

In the 1680s, Newton turned his attention to forces and motion. He worked on applying his three laws of motion to orbiting bodies, projectiles, pendulums, and free-fall situations. This work led him to formulate his famous law of universal gravitation.

According to legend, Newton thought of the idea while sitting in his Lincolnshire garden. He watched an apple fall from a tree. He wondered if the same force that caused the apple to fall toward the center of Earth (gravity) might be responsible for keeping the moon in orbit around Earth, and the planets in orbit around the sun.

This concept was truly revolutionary. Less than 50 years earlier, it was commonly believed that some sort of invisible shield held the planets in orbit.

Important contributor in spite of conflict

In 1687, Newton published his ideas in a famous work known as the *Principia*. He jealously guarded the work as entirely his. He bitterly resented the suggestion that he should acknowledge the exchange of ideas with other scientists (especially Hooke) as he worked on his treatise.

Newton left Cambridge to take a government position in London in 1696. His years of active scientific research were over. However, almost three centuries after his death in 1727, Newton remains one of the most important contributors to our understanding of how the universe works.

Reading reflection

1. Important phases of Newton's education and scientific work occurred in isolation. Why might this have been helpful to him? On the other hand, why is working in isolation problematic for developing scientific ideas?

2. Newton began his academic career in 1667. For how long was he a working scientist? Was he a very productive scientist? Justify your answer.

3. Briefly state one of Newton's three laws of motion in your own words. Give an explanation of how this law works.

4. Define the law of universal gravitation in your own words.

5. The orbit of a space shuttle is surprisingly like an apple falling from a tree to Earth. The space shuttle is simply moving so fast that the path of its fall is an orbit around our planet. Which of Newton's laws helps explain the orbit of a space shuttle around Earth and the orbit of Earth around the sun?

6. **Research:** Newton was outraged when, in 1684, German mathematician Wilhelm Leibniz published a calculus book. Find out why, and describe how the issue is generally resolved today.

Name: ______________________ Date: __________

CONTENT 6.2

6.2 Newton's Second Law

READ

- Newton's second law states that the acceleration of an object is directly related to the force on it, and inversely related to the mass of the object. You need more force to move or stop an object with a lot of mass (or *inertia*) than you need for an object with less mass.
- The formula for the second law of motion (first row below) can be rearranged to solve for mass and force.

What do you want to know?	What do you know?	The formula you will use
acceleration (a)	force (F) and mass (m)	$\text{acceleration} = \dfrac{\text{force}}{\text{mass}}$
mass (m)	acceleration (a) and force (F)	$\text{mass} = \dfrac{\text{force}}{\text{acceleration}}$
force (F)	acceleration (a) and mass (m)	$\text{force} = \text{acceleration} \times \text{mass}$

EXAMPLE

- How much force is needed to accelerate a truck with a mass of 2,000 kilograms at a rate of 3 m/s^2?

$$F = m \times a = 2{,}000 \text{ kg} \times 3 \text{ m/s}^2 = 6{,}000 \text{ kg-m/s}^2 = 6{,}000 \text{ N}$$

- What is the mass of an object that requires 15 N to accelerate it at a rate of 1.5 m/s^2?

$$m = \frac{F}{a} = \frac{15 \text{ N}}{1.5 \text{ m/s}^2} = \frac{15 \text{ kg-m/s}^2}{1.5 \text{ m/s}^2} = 10 \text{ kg}$$

PRACTICE

1. What is the acceleration of a 2,000.-kilogram truck if a force of 4,200. N is used to make it start moving forward?
2. What is the acceleration of a 0.30-kilogram ball that is hit with a force of 25 N?
3. How much force is needed to accelerate a 68-kilogram skier at 1.2 m/s^2?
4. What is the mass of an object that requires a force of 30 N to accelerate at 5 m/s^2?
5. What is the force on a 1,000.-kilogram elevator that is falling freely under the acceleration of gravity only?
6. What is the mass of an object that needs a force of 4,500 N to accelerate it at a rate of 5 m/s^2?
7. What is the acceleration of a 6.4-kilogram bowling ball if a force of 12 N is applied to it?

Name: ______________________________ Date: __________

CONTENT 6.3

6.3 Applying Newton's Laws of Motion

READ

In the second column of the table below, write each of Newton's three laws of motion. Use your own wording. In the third column of the table, describe an example of each law. To find examples of Newton's laws, think about all the activities you do in one day.

Newton's laws of motion	Write the law here in your own words	Example of the law
The first law		
The second law		
The third law		

PRACTICE

1. When Jane drives to work, she always places her pocketbook on the passenger's seat. By the time she gets to work, her pocketbook has fallen on the floor in front of the passenger seat. One day, she asks you to explain why this happens in terms of physical science. What do you say?

2. You are waiting in line to use the diving board at your local pool. While watching people dive into the pool from the board, you realize that using a diving board to spring into the air before a dive is a good example of Newton's third law of motion. Explain how a diving board illustrates Newton's third law of motion.

3. You know the mass of an object and the force applied to the object to make it move. Which of Newton's laws of motion will help you calculate the acceleration of the object?

4. How many newtons of force are represented by the following amount: $3\ \text{kg} \cdot \text{m/s}^2$?
   Select the correct answer (a, b, or c) and justify your answer.

   a. 6 newtons　　b. 3 newtons　　c. 1 newton

5. Your shopping cart has a mass of 65 kilograms. In order to accelerate the shopping cart down an aisle at $0.30\ \text{m/s}^2$, what force would you need to use or apply to the cart?

6. A small child has a wagon with a mass of 10 kilograms. The child pulls on the wagon with a force of 2 newtons. What is the acceleration of the wagon?

7. You dribble a basketball while walking on a basketball court. List and describe the pairs of action-reaction forces in this situation.

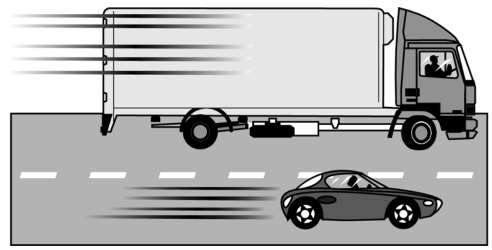
8. Pretend that there is no friction at all between a pair of ice skates and an ice rink. If a hockey player using this special pair of ice skates was gliding along on the ice at a constant speed and direction, what would be required for him to stop?

Name: ____________________ Date: __________

6.3 Momentum

READ

Which is more difficult to stop: A tractor-trailer truck barreling down the highway at 35 meters per second, or a small two-seater sports car traveling the same speed?



You probably guessed that it takes more force to stop a large truck than a small car. In physics terms, we say that the truck has greater *momentum*.

We can find momentum using this equation:

$$\text{momentum} = \text{mass of object} \times \text{velocity of object}$$

Velocity is a term that refers to both speed and direction. For our purposes we will assume that the vehicles are traveling in a straight line. In that case, velocity and speed are the same.

The equation for momentum is abbreviated like this: $P = m \times v$.

Momentum, symbolized with a *P*, is expressed in units of kg · m/s; *m* is the mass of the object, in kilograms; and *v* is the velocity of the object in m/s.

PRACTICE

Use your knowledge about solving equations to work out the following problems:

1. If the truck has a mass of 4,000. kilograms, what is its momentum? Express your answer in kg · m/s.
2. If the car has a mass of 1,000. kilograms, what is its momentum?
3. An 8-kilogram bowling ball is rolling in a straight line toward you. If its momentum is 16 kg · m/s, how fast is it traveling?
4. A beach ball is rolling in a straight line toward you at a speed of 0.5 m/s. Its momentum is 0.25 kg · m/s. What is the mass of the beach ball?
5. A 4,500.-kilogram truck travels in a straight line at 10. m/s. What is its momentum?
6. A 1,500.-kilogram car is also traveling in a straight line. Its momentum is equal to that of the truck in the previous question. What is the velocity of the car?
7. Which would take more force to stop in 10. seconds: an 8.0-kilogram ball rolling in a straight line at a speed of 0.2 m/s or a 4.0-kilogram ball rolling along the same path at a speed of 1.0 m/s?
8. The momentum of a car traveling in a straight line at 25 m/s is 24,500 kg·m/s. What is the car's mass?
9. A 0.14-kilogram baseball is thrown in a straight line at a velocity of 30. m/s. What is the momentum of the baseball?
10. Another pitcher throws the same baseball in a straight line. Its momentum is 2.1 kg · m/s. What is the velocity of the ball?
11. A 1-kilogram turtle crawls in a straight line at a speed of 0.01 m/s. What is the turtle's momentum?

Name: ______________________ Date: __________

CONTENT 6.3

6.3 Momentum Conservation

READ

Just as forces are equal and opposite (according to Newton's third law), changes in momentum are also equal and opposite. This is because when objects exert forces on each other, their motion is affected.

The law of momentum conservation states that if interacting objects in a system are not acted on by outside forces, the total amount of momentum in the system cannot change.

The formula below can be used to find the new velocities of objects if both keep moving after the collision.

$$\text{total momentum of a system before} = \text{total momentum of a system after}$$
$$m_1v_{1(\text{ initial})} + m_2v_{2\ (\text{initial})} = m_1v_{3\ (\text{final})} + m_2v_{4\ (\text{final})}$$

If two objects are initially at rest, the total momentum of the system is zero.

$$\text{the momentum of a system before a collision} = 0$$

For the final momentum to be zero, the objects must have equal momenta in opposite directions.

$$0 = \text{the momentum of a system after a collision}$$
$$0 = m_1v_3 + m_2v_4$$
$$m_1v_3 = -(m_2v_4)$$

EXAMPLE

Example 1: What is the momentum of a 0.2-kilogram steel ball that is rolling at a velocity of 3.0 m/s?

$$\text{momentum} = m \times v = 0.2\text{ kg} \times \frac{3\text{ m}}{\text{s}} = 0.6\text{ kg} \cdot \frac{m}{\text{s}}$$

Example 2: You and a friend stand facing each other on ice skates. Your mass is 50. kilograms and your friend's mass is 60. kilograms. As the two of you push off each other, you move with a velocity of 4.0 m/s to the right. What is your friend's velocity?

Looking for Your friend's velocity to the left.	**Solution** $m_1v_3 = -(m_2v_4)$ $(50.\text{ kg})(4.0\text{ m/s}) = -(60.\text{ kg})(v_4)$ $\frac{200\text{ kg-m/s}}{-(60\text{ kg})} = v_4$ $-3.3\text{ m/s} = v_4$ Your friend's velocity to the left is 3.3 m/s.
Given Your mass of 50. kg. Your friend's mass of 60. kg. Your velocity of 4.0 m/s to the right.	
Relationship $m_1v_3 = -(m_2v_4)$	

PRACTICE

1. If a ball is rolling at a velocity of 1.5 m/s and has a momentum of 10.0 kg·m/s, what is the mass of the ball?
2. What is the velocity of an object that has a mass of 2.5 kg and a momentum of 1,000 kg · m/s?
3. A pro golfer hits 45.0-gram golf ball, giving it a speed of 75.0 m/s. What momentum has the golfer given to the ball?
4. A 400-kilogram cannon fires a 10-kilogram cannonball at 20 m/s. If the cannon is on wheels, at what velocity does it move backward? (This backward motion is called recoil velocity.)
5. Eli stands on a skateboard at rest and throws a 0.5-kg rock at a velocity of 10.0 m/s. Eli moves back at 0.05 m/s. What is the combined mass of Eli and the skateboard?
6. As the boat in which he is riding approaches a dock at 3.0 m/s, Jasper stands up in the boat and jumps toward the dock. Jasper applies an average force of 800. newtons on the boat for 0.30 seconds as he jumps.
   a. How much momentum does Jasper's 80.-kilogram body have as it lands on the dock?
   b. What is Jasper's speed on the dock?
7. Daryl the delivery guy gets out of his pizza delivery truck but forgets to set the parking brake. The 2,000.-kilogram truck rolls down hill reaching a speed of 30 m/s just before hitting a large oak tree. The vehicle stops 0.72 s after first making contact with the tree.
   a. How much momentum does the truck have just before hitting the tree?
   b. What is the average force applied by the tree?
8. Two billion people jump up in the air at the same time with an average velocity of 7.0 m/s. If the mass of an average person is 60 kilograms and the mass of Earth is 5.98×10^{24} kilograms:
   a. What is the total momentum of the two billion people?
   b. What is the effect of their action on Earth?
9. Tammy, a lifeguard, spots a swimmer struggling in the surf and jumps from her lifeguard chair to the sand beach. She makes contact with the sand at a speed of 6.00 m/s, leaving an indentation in the sand 0.10 m deep.
   a. If Tammy's mass is 60. kilograms, what is the momentum as she first touches the sand?
   b. What is the average force applied on Tammy by the sand beach?
10. When a gun is fired, the shooter describes the sensation of the gun kicking. Explain this in terms of momentum conservation.
11. What does it mean to say that momentum is conserved?

Name: ______________________________ Date: __________

CONTENT 6.3

6.3 Collisions and Conservation of Momentum

READ

The law of conservation of momentum tells us that as long as colliding objects are not influenced by outside forces like friction, the total amount of momentum in the system before and after the collision is the same.

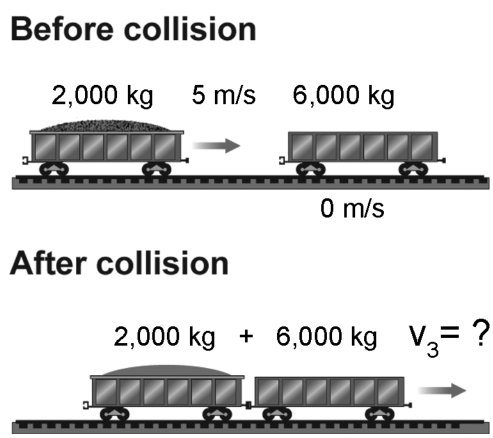
We can use the law of conservation of momentum to predict how two objects will move after a collision. Use the problem solving steps and the examples below to help you solve collision problems.

Problem Solving Steps

1. Draw a diagram.
2. Assign variables to represent the masses and velocities of the objects before and after the collision.
3. Write an equation stating that the total momentum before the collision equals the total after.
4. Plug in the information that you know.
5. Solve your equation.

EXAMPLE

A 2,000-kilogram railroad car moving at 5 m/s collides with a 6,000-kilogram railroad car at rest. If the cars coupled together, what is their velocity after the inelastic collision?



Looking for

m_3 = the velocity of the combined railroad cars after an inelastic collision

Given

Initial speed and mass of both cars:
$m_1 = 2{,}000$ kg, $v_1 = 5$ m/s
$m_2 = 6{,}000$ kg, $v_2 = 0$ m/s
Combined mass of the two cars:
$m_1 + m_2 = 8{,}000$ kg

Relationship

$m_1v_1 + m_2v_2 = (m_1 + m_2)m_3$

Solution

$$(2000\text{ kg})(5\text{ m/s}) + (6000\text{ kg})(0\text{ m/s}) = (2000\text{ kg} + 6000\text{ kg})v_3$$
$$10{,}000\text{ kg-m/s} = (8000\text{ kg})v_3$$
$$\frac{10{,}000\text{ kg-m/s}}{8000\text{ kg}} = v_3$$
$$10\text{ m/s} = v_3$$

The velocity of the two combined cars after the collision is 10 m/s.

PRACTICE

1. What is the momentum of a 100.-kilogram fullback carrying a football on a play at a velocity of 3.5 m/s?
2. What is the momentum of a 75.0-kilogram defensive back chasing the fullback at a velocity of 5.00 m/s?
3. A 2,000-kilogram railroad car moving at 5 m/s to the east collides with a 6,000-kilogram railroad car moving at 3 m/s to the west. If the cars couple together, what is their velocity after the collision?
4. A 4.0-kilogram ball moving at 8.0 m/s to the right collides with a 1.0-kilogram ball at rest. After the collision, the 4.0-kilogram ball moves at 4.8 m/s to the right. What is the velocity of the 1-kilogram ball?
5. A 0.0010-kg pellet is fired at a speed of 50.0m/s at a motionless 0.35-kg piece of balsa wood. When the pellet hits the wood, it sticks in the wood and they slide off together. With what speed do they slide?
6. Terry, a 70.-kilogram tailback, runs through his offensive line at a speed of 7.0 m/s. Jared, a 100-kilogram linebacker, running in the opposite direction at 6.0 m/s, meets Jared head-on and “wraps him up.” What is the result of this tackle?
7. Snowboarding cautiously down a steep slope at a speed of 7.0 m/s, Sarah, whose mass is 50. kilograms, is afraid she won't have enough speed to travel up a slight uphill grade ahead of her. She extends her hand as her friend Trevor, who has a mass of 100. kilograms, is about to pass her traveling at 16 m/s. If Trevor grabs her hand, calculate the speed at which the friends will be sliding.
8. Tex, an 85.0 kilogram rodeo bull rider is thrown from the bull after a short ride. The 520. kilogram bull chases after Tex at 13.0 m/s. While running away at 3.00 m/s, Tex jumps onto the back of the bull to avoid being trampled. How fast does the bull run with Tex aboard?
9. Identical twins Kate and Karen each have a mass of 45 kg. They are rowing their boat on a hot summer afternoon when they decide to go for a swim. Kate jumps off the front of the boat at a speed of 3.00 m/s. Karen jumps off the back at a speed of 4.00 m/s. If the 70.-kilogram rowboat is moving at 1.00 m/s when the girls jump, what is the speed of the rowboat after the girls jump?
10. A 0.10-kilogram piece of modeling clay is tossed at a motionless 0.10-kilogram block of wood and sticks. The block slides across a frictionless table at 15 m/s.
    a. At what speed was the clay tossed?
    b. The clay is replaced with a “bouncy” ball tossed with the same speed. The bouncy ball rebounds from the wooden block at a speed of 10 m/s. What effect does this have on the wooden block? Why?

Name: ______________________________ Date: __________

CHALLENGE! 6.3

6.3 Rate of Change of Momentum

READ

Momentum is given by the expression $p = mv$ where p is the momentum of an object of mass m moving with velocity v. The units of momentum are *kg-m/s*. Change of momentum (represented Δp) over a time interval (represented Δt) is also called the rate of change of momentum.

Since, momentum is $p = mv$, if the mass remains constant during the time Δt, then:

$$\frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t}$$

The expression, $\frac{\Delta v}{\Delta t}$, represents change in velocity over change in time, also known as acceleration. From Newton's second law, we know that acceleration equals force divided by mass ($a = F/m$). Rearranging the equation, we see that force equals mass times acceleration ($F = ma$). Similarly, force (F) equals change in momentum over change in time.

$$F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$$

A mass, m, moving with velocity, v, has momentum mv. If this momentum becomes zero over some change in time (Δt), then there is a force, $F = (mv - 0)/\Delta t$.

- mv is the initial momentum.
- 0 is the momentum after a change in time Δt.

When a car accelerates or decelerates, we feel a force that pushes back during acceleration and pushes us forward during deceleration. When the car brakes slowly, the force is small. However, when the car brakes quickly, the force increases considerably.

EXAMPLE

Example 1. An 80-kg woman is a passenger in a car going 90 km/h. The driver puts on the brakes and the car comes to a stop in 2 seconds. What is the average force felt by the passenger?

First, convert the velocity to a value that is in meters per second: 90 km/h = 25 m/s. Next, use the equation that relates force and momentum:

$$\text{Force} = \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t} = 80\text{ kg}\frac{(25 - 0)\text{ m/s}}{2\text{ s}} = 1{,}000\text{ N}$$

This is a large force, and for the passenger to stay in her seat, she must be strapped in with a seat belt.

When the stopping time decreases from 2 seconds to 1 second, the force increases to 2,000 newtons. When the car is involved in a crash, the change in momentum happens over a much shorter period of time, thereby creating very large forces on the passenger. Air bags and seat belts help by slowing down the person's momentum change, resulting in smaller forces and a reduced chance for injury. Let's look at some numbers.

The car travels at 90 kilometers per hour, crashes, and comes to a stop in 0.1 seconds. The air bag inflates and cushions the person for 1.5 seconds. Let's calculate the force experienced by the passenger in an automobile without air bags and in one case with air bags.

- Without the air bag, the momentum change happens over 0.1 seconds. This results in a force:

$$\text{Force} = 80\ \text{kg}\frac{25\ \text{m/s}}{0.1\ \text{s}} = 2{,}000\ \text{N}$$

  The human body is not likely to survive a force as large as this.

- With the air bag, the force created is:

$$\text{Force} = 80\ \text{kg}\frac{25\ \text{m/s}}{1.5\ \text{s}} = 1{,}333\ \text{N}$$

  The chances for survival are much higher.

Example 2. A pile is driven into the ground by hitting it repeatedly. If the pile is hit by the driver mass at a rate of 100 kg/s and with a speed of 10 m/s, calculate the resulting average force on the pile.

We are told that the driver mass hits the pile at a rate of 100 kg/s. What does this mean exactly? We can have a 100-kilogram mass hitting the pile every second, or a 50-kilogram mass hitting the pile every half-second, or a 200-kilogram mass hitting the pile every 2 seconds. You get the idea.

The speed (v) with which the mass hits the pile is 10 m/s. The mass (m) is 100 kilograms. Time changes occur at 1-second intervals. The force on the pile is:

$$\text{Force} = m\frac{\Delta v}{\Delta t} = 100\ \frac{\text{kg}}{\text{s}}\ 10\ \frac{\text{m}}{\text{s}} = 1{,}000\ \frac{\text{kg m}}{\text{s}^2} = 1{,}000\ \text{N}$$

PRACTICE

1. A 1,000-kg wrecking ball hits a wall with a speed of 2 m/s and comes to a stop in 0.01 s. Calculate the force experienced by the wall.
2. A 0.15-kg soccer ball is rolling with a speed of 10 m/s and is stopped by the frictional force between it and the grass. If the average friction force is 0.5 N, how long would this take?
3. Water comes out of a fire hose at a rate of 5.0 kg/s and with a speed of 50 m/s. Calculate the force on the hose. (This is the force that the firefighter has to provide in order to hold the hose.)
4. Water from a fire hose is hitting a wall straight on. The water comes out with a flow rate of 25 kg/s and hits the wall with a speed of 30. m/s. What is the resulting force exerted on the wall by the water?
5. The water at Niagara Falls flows at a rate of 3.0 million kg/s. The water hits the bottom of the falls at a speed of 25 m/s. What is the force generated by the change in momentum of the falling water?

CHALLENGE!

6.3

6. A 50.-g (0.050-kg) egg that is dropped from a height of 5.0 m will hit the floor with a speed of about 10. m/s. The hard floor forces the egg to stop very quickly. Let's say that it will stop in 0.0010 second.

a. What is the force created on the egg?

b. The egg will break at the force you calculated for 6(a). Imagine that a 50.-kilogram person fell down on the egg falling under the influence of gravity. What would the force of the person on the egg be?

c. Do you think the egg will break if the person fell on it? Why or why not?

d. If we now drop the egg onto a pillow, it will allow the egg to stop over a much longer time compared with the time it takes for it to stop on the hard surface. The weight and the velocity of the egg is still the same, but now the time it takes for the egg to come to rest is much longer, about 0.5 second or about 500 times longer than the time it took to stop on the floor. What would the force on the egg be under these circumstances?

e. Do you think the egg will break when it drops on the pillow? Why or why not?