

1.3 Dimensional Analysis

READ


Dimensional analysis is a way to find the correct label (also called units or dimensions) for the solution to a problem. In dimensional analysis, we treat the units the same way that we treat the numbers. For example, this problem shows how you can “cancel” the sevens and then perform the multiplication:

$$\frac{3}{\cancel{7}} \cdot \frac{\cancel{7}}{8} = \frac{3}{8}$$

In some problems, there are no numerical cancellations to make, but you need to pay close attention to the units (or dimensions):

- If there are 16 ounces in one pound, how many ounces are in four pounds?

$$\frac{16 \text{ oz}}{1 \text{ lb}} \cdot 4 \text{ lb} = \frac{16 \cdot 4 \text{ oz} \cdot \cancel{\text{lb}}}{1 \cancel{\text{lb}}} = \frac{64 \text{ oz}}{1} = 64 \text{ oz}$$

The “lbs” may be cancelled either before or after the multiplication.

The goal of dimensional analysis is to simplify a problem by focusing on the units of measurement (dimensions).

Dimensional analysis is very useful when converting between units (like converting inches to yards, or converting between the English and SI systems of measurement).

EXAMPLE

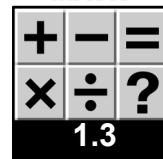
- The ninth grade class is having a reward lunch for collecting the most food for a canned food drive. They have decided to order pizza. They are figuring two slices of pizza per student. Each pizza that will be ordered will have 12 slices. There are 220 students total in the ninth grade. How many pizzas should they order?

Solution:

- Determine what we want to find out: here, it is the number of whole pizzas needed to feed 220 ninth graders. It's important to remember that if the solution is to have the label “pizzas,” “pizzas” should be kept in the numerator as the problem is set up.
- Determine what we know. We know that they're planning 2 slices of pizza per student, that there are 12 slices in each pizza, and that there are 220 ninth graders.
- Write what you know as fractions with units. Here, we have: $\frac{2 \text{ slices}}{\text{student}}$, $\frac{12 \text{ slices}}{\text{pizza}}$ and 220 students.

Notice that in the fraction, $\frac{12 \text{ slices}}{\text{pizza}}$, “pizza” is in the denominator.

Recall that (from step #1, above) “pizza” should be kept in the numerator, as it will be the label of the final solution. To correct this problem, just switch the numerator and denominator: $\frac{1 \text{ pizza}}{12 \text{ slices}}$.



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4. Set up the problem by focusing on the units. Just writing the information as a multiplication problem, we have:

$$\frac{1 \text{ pizza}}{12 \text{ slices}} \cdot \frac{2 \text{ slices}}{\text{student}} \cdot \frac{220 \text{ students}}{1}$$

5. Calculate: $\frac{1 \text{ pizza}}{12 \text{ slices}} \cdot \frac{2 \text{ slices}}{\text{student}} \cdot \frac{220 \text{ students}}{1} = \frac{110 \text{ pizza} \cdot \cancel{\text{slices}} \cdot \cancel{\text{students}}}{3 \cancel{\text{ slices}} \cdot \cancel{\text{ students}}} = 36\frac{2}{3}$ pizzas

Therefore, 37 pizzas will need to be ordered.

Notice that canceling the units can be done either before or after the multiplication.

6. Check your solution for reasonableness: Since there are 12 slices in each pizza, and we're figuring that each student will eat 2 slices, one pizza will feed 6 students. It is expected that a little less than 40 pizzas would be needed. It does seem reasonable that 37 pizzas would feed 220 students.

PRACTICE



- Multiply. Be sure to label your answers.
 - $\frac{\$12.00}{1 \text{ hr}} \cdot \frac{6 \text{ hr}}{1 \text{ day}}$
 - $\frac{2 \text{ lbs}}{1 \text{ person}} \cdot \frac{7 \text{ days}}{1 \text{ week}} \cdot \frac{15 \text{ people}}{1 \text{ day}}$
- Use dimensional analysis to convert each. You may need to use a reference to find some conversion factors. Show all of your work.
 - 11.0 quarts to some number of gallons
 220. centimeters to some number of meters
 6000. inches to some number of miles
 - How many cups are there in 4.0 gallons?
- Use dimensional analysis to find each solution. You may need to use a reference to find some conversion factors. Show all of your work.
 - Frank just graduated from eighth grade. Assuming exactly four years from now he will graduate from high school, how many seconds does he have until his high school graduation?
 - In 2005, Christian Cantwell won the US outdoor track and field championship shot put competition with a throw of 21.64 meters. How far is this in feet?
 - A recipe for caramel oatmeal cookies calls for 1.5 cups of milk. Sam is helping to make the cookies for the soccer and football teams plus the cheerleaders and marching band, and needs to multiply the recipe by twelve. How much milk (in quarts) will he need altogether?
 - How many football fields (including the 10 yards in each end zone) would it take to make a mile?
 - Corey's sister's car gets 30. miles on each gallon of gas. How many kilometers per gallon is this?
 - Convert your answer from (e) to kilometers per liter.
 - A car is traveling at a rate of 65 miles per hour. How many feet per second is this?