

4.1 Solving Equations with One Variable

READ



It is useful to know formulas for calculating different quantities. Often, the formulas are very straightforward. It's easy to calculate the volume of a rectangular solid when you know the formula:

$$\text{Volume} = V = \text{length} \times \text{width} \times \text{height} \quad (V = l \times w \times h)$$

and the length, width, and height of the solid. It's a little more challenging when you know the volume, length, and width, but need to find the height. It then becomes necessary to solve an equation in order to determine the unknown (in this case, the height).

EXAMPLES



1. The volume of a rectangular solid, with a length of 1.5 cm, is 10.98 cm^3 . The width of the same solid is 1.2 cm. Find its height.

Explanation/Answer: use the formula $V = l \times w \times h$, and then plug in what is known, leaving the variable (h) for the unknown. Solve the equation for h to find the height.

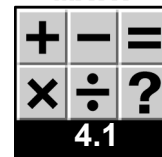
The Work:	What's happening:
$V = l \times w \times h$	Formula
$10.98 \text{ cm}^3 = 1.5 \text{ cm} \times 1.2 \text{ cm} \times h$	Plug in known values.
$10.98 \text{ cm}^3 = 1.8 \text{ cm}^2 \times h$	Complete arithmetic, multiply 1.5×1.2
$\frac{10.98 \text{ cm}^3}{1.8 \text{ cm}^2} = \frac{1.8 \text{ cm}^2 \times h}{1.8 \text{ cm}^2}$	Divide both sides by 1.8 cm^2 , to get h alone.
$6.1 \text{ cm} = 1 \times h$	Do the division; $10.98 \text{ cm}^3 \div 1.8 \text{ cm}^2 = 6.1 \text{ cm}$, $1.8 \text{ cm}^2 \div 1.8 \text{ cm}^2 = 1$
$6.1 \text{ cm} = h$	

Check the Work:

$V = l \times w \times h$	
$V = 1.5 \text{ cm} \times 1.2 \text{ cm} \times 6.1 \text{ cm}$	Multiply $1.5 \text{ cm} \times 1.2 \text{ cm} \times 6.1 \text{ cm}$. If the answer is 10.98 cm^3 , the solution, $h = 6.1 \text{ cm}$, is correct.
$1.5 \text{ cm} \times 1.2 \text{ cm} \times 6.1 \text{ cm} = 10.98 \text{ cm}^3$	The product does equal 10.98 cm^3 , the solution is correct.

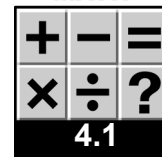
In summary:

The height (h) of a rectangular solid whose volume is 10.98 cm^3 , whose length is 1.5 cm, and whose width is 1.2 cm, is 6.1 cm.



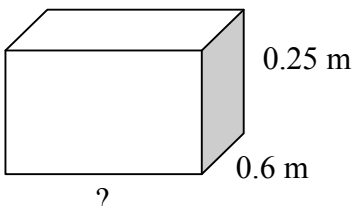
2. The density of titanium is 4.5 g/cm^3 . A titanium pendant's mass is 2.25 grams. Use the formula $\text{Density} = \frac{\text{mass}}{\text{volume}}$, or $D = \frac{m}{v}$, or to find its volume.

The Work:	What's happening:
$D = \frac{m}{v}$	Formula
$4.5 \text{ g/cm}^3 = \frac{2.25 \text{ g}}{V}$	Plug in known values.
$\frac{4.5 \text{ g}}{1 \text{ cm}^3} = \frac{2.25 \text{ g}}{V}$	Rewrite 4.5 g/cm^3 as $\frac{4.5 \text{ g}}{1 \text{ cm}^3}$
$4.5 \text{ g} \times V = (2.25 \text{ g}) \times (1 \text{ cm}^3)$	Think of $\frac{4.5 \text{ g}}{1 \text{ cm}^3} = \frac{2.25 \text{ g}}{V}$ as a proportion. Then set the cross products equal
$4.5 \text{ g} \times V = 2.25 \text{ g} \times 1 \text{ cm}^3$	Do arithmetic: $2.25 \text{ g} \times 1 \text{ cm}^3 = 2.25 \text{ g} \times 1 \text{ cm}^3$
$\frac{4.5 \text{ g} \times V}{4.5 \text{ g}} = \frac{2.25 \text{ g} \times 1 \text{ cm}^3}{4.5 \text{ g}}$	Divide both sides of the equation by 4.5 g to get V alone.
$V = 0.5 \text{ cm}^3$	Do the division on each side. Remember to cancel units as well as divide the numbers.
Check the Work:	
$D = \frac{m}{v}$	
$4.5 \text{ g/cm}^3 = \frac{2.25 \text{ g}}{0.5 \text{ cm}^3}$	Divide $2.25 \div 0.5$. If the answer is 4.5 g/cm^3 , the solution, $V = 0.5 \text{ cm}^3$, is correct.
$2.25 \div 0.5 = 4.5 \text{ g/cm}^3$	The quotient does equal 4.5 g/cm^3 ; therefore, the solution is correct.
In summary:	
The volume of a titanium pendant whose mass is 2.25 grams is 4.5 g/cm^3 .	

**PRACTICE**

Use the formula $V = l \times w \times h$ to set up and solve for the unknown in each.

- Find the width (w) of a rectangular solid whose length is 12 mm, and whose height is 15 mm, if the volume of the solid is 720 mm^3 .
- Find the length of this rectangular solid whose volume is 0.12 m^3 .



- The length and width of a rectangular solid are 2.15 cm. Its volume is 36.98 cm^3 . Find the height of this rectangular solid.

Use the formula $\text{Speed} = \frac{\text{distance}}{\text{time}}$, or $S = \frac{d}{t}$ to set up and solve for the unknown in each.

Here, speed is measured in meters/second (m/s), distance is measured in meters (m), and time is measured in seconds (s).

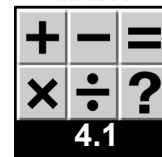
- How far will a marble rolling at a speed of 0.25 m/s travel in 30. seconds?
- Nate throws a paper wad to Ali who is sitting exactly 1.8 meters away. The paper wad was only in the air for 0.45 seconds. How fast was it traveling?
- How long does it take a battery operated toy car to travel 3 meters at a speed of 0.1 m/s?
- A dog is running 3.20 m/s. How long will it take him to go 100. meters?

Use the formula $\text{Density} = \frac{\text{mass}}{\text{volume}}$, or $D = \frac{m}{v}$, to set up and solve for the unknown in each.

Here, density (D) is measured in grams per cubic centimeter (g/cm^3), mass (m) is measured in grams (g), and volume (V) is measured in cubic centimeters (cm^3).

- What is the density of a steel nail whose volume is 3.2 cm^3 and whose mass is 25 g?
- Find the mass of a cork whose density is 0.12 g/cm^3 and whose volume is 9.0 cm^3 ?
- An ice cube's volume is 4.9 cm^3 . Find its mass if its density is 0.92 g/cm^3 .
- A solid plastic ball's mass is 225 g. The density of the plastic is 2.00 g/cm^3 . What is the volume of the ball?
- Find the volume of an ice cube whose mass is 2.08 g. See question #10 for the density of ice.

Use the formula: $\text{Force} = \text{pressure} \times \text{area}$ to set up and solve for each unknown.



Here, force is measured in Newtons (N), pressure is measured in Pascals (Pa), and area is measured in square meters (m²). Hint: 1 Pa = 1 N / m².

EXAMPLE

- A drinking glass is sitting on the kitchen table. The glass has a weight of 2 N. Its base has an area of 0.005 m². How much pressure does the drinking glass exert on the table?

Explanation/Answer:

The Work:	What's happening:
Force = pressure \times area	Formula
$2 \text{ N} = p \times 0.005 \text{ m}^2$	Plug in known values.
$\frac{2 \text{ N}}{0.005 \text{ m}^2} = \frac{p \times 0.005 \text{ m}^2}{0.005 \text{ m}^2}$	Divide both sides by 0.005 m ² to get p alone.
$400 \text{ N/m}^2 = p \times 1$	Do the division: $2 \text{ N} \div 0.005 \text{ m}^2 = 400 \text{ N / m}^2$, $0.005 \text{ m}^2 \div 0.005 \text{ m}^2 = 1$
$400 \text{ Pa} = p$	Rewrite 400 N/m ² as 400 Pa, multiply; $p \times 1 = p$.
In summary:	
A drinking glass with a weight of 2 N and whose base has area 0.005 m ² exerts 400 Pa of pressure on the table it sits on.	

PRACTICE

- A tea kettle's base has an area of 0.008 m². It puts 1,000 Pascals of pressure on the stove where it sits. What is the weight of the kettle?
- A block of wood whose base has an area of 4 m² has a weight of 80 N. How much pressure does the block place on the floor on which it sits?
- A sculpture's base has an area of 2.50 m². How much pressure does the sculpture place on the wooden display case where it sits, if it has a weight of 540. N?
- A student is breaking class rules by standing on a chair. If her feet have a total area of 0.04 m², and her weight is 600. N, how much pressure is she putting on the chair?