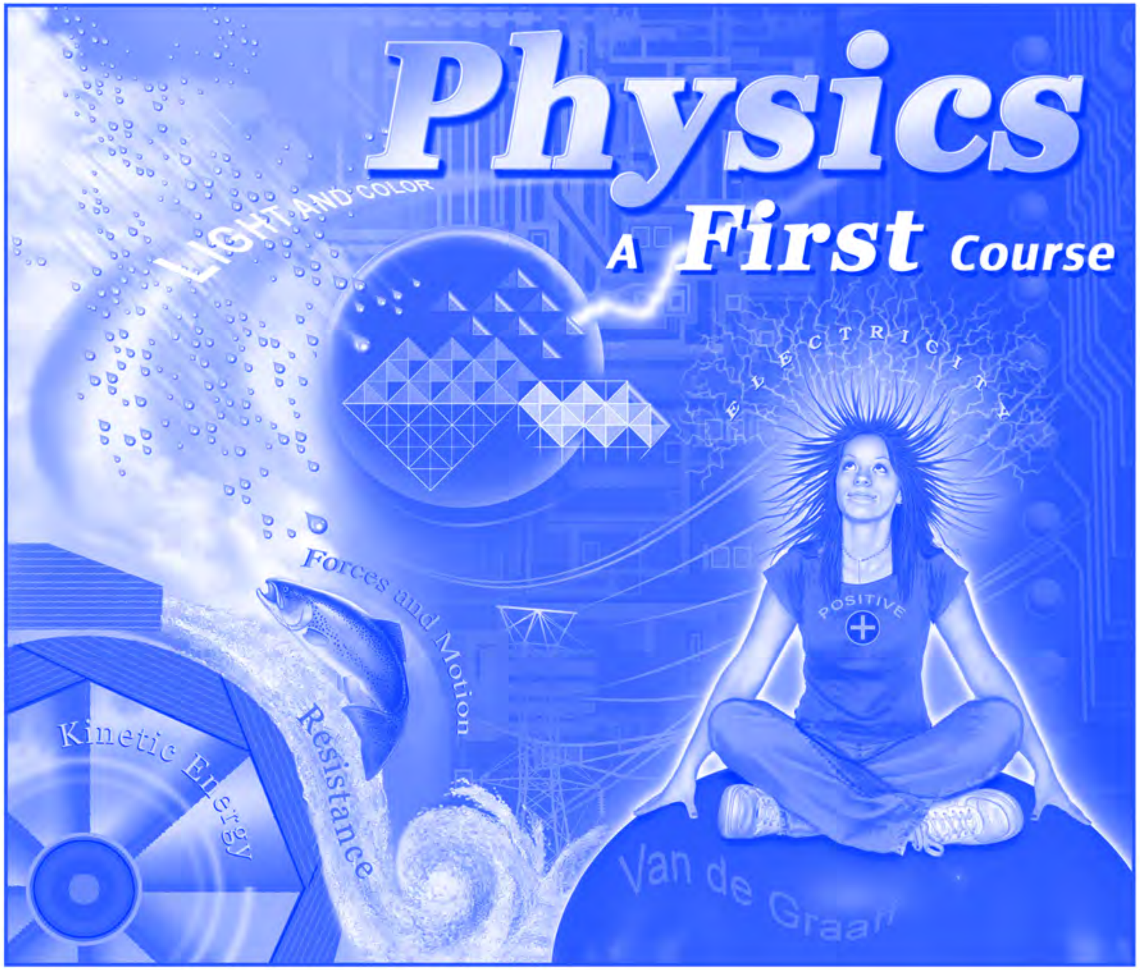

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Chapter 19: Harmonic Motion

The forward motion of a pedaling cyclist is an example of linear motion. The pedaling action and turning of the cycle's wheels are examples of harmonic motion. Harmonic motion is motion that repeats. A system with harmonic motion is an oscillator. A cycle is the individual unit of harmonic motion that is repeated. The time it takes to complete one cycle is called the period. A unit of one cycle per second is called a hertz (Hz). The amplitude describes the "size" of a cycle. With a pendulum, the amplitude is often a distance or angle. With other types of oscillators the amplitude might be voltage or pressure. The amplitude of harmonic motion can be represented using a harmonic motion graph.

Systems that have harmonic motion always move back and forth around an equilibrium position. Equilibrium is maintained by restoring forces. A restoring force is any force that acts to pull the system back toward equilibrium. For a pendulum the equilibrium position is the center. A restoring force of gravity always pulls the pendulum towards the center. The pendulum overshoots the center because of its own inertia. This cycle repeats itself over and over.

Investigation Vocabulary

harmonic motion, cycle, amplitude, oscillator, hertz, period, frequency, restoring force

Investigation 19A: Harmonic Motion

Activities

- Measure the amplitude of a pendulum.
- Measure the cycle of a pendulum.
- Identify the variables that affect a pendulum.
- Sketch harmonic motion graphs.

Key Question How does physics describe the repeating motion of a pendulum or swing?

Time One class period

Comments Set up the Pendulum apparatus and become familiar with its operation prior to teaching the investigation.

Investigation 19B: Natural Frequency and Resonance

Activities

- Make an oscillator.
- Measure the natural frequency of an oscillator.
- Identify variables that affect an oscillator.
- Identify the relationship between frequency, mass and force.

Key Question How do force and mass affect harmonic motion?

Time One class period

Comments You will need steel washers for this investigation. These may be purchased at any hardware store. Washers that fit 1/2 inch bolts are about the right size and mass.

Setup Information

Setup and Materials A

Students use the Pendulum apparatus for this investigation.

Students work in groups of three to five.

Each group should have:

- Physics stand
- Pendulum apparatus
- Timer and 1 photogate
- Graph paper

Setup and Materials B

Students work in groups of three to five.

Each group should have:

- Physics stand
- Timer and 1 photogate
- Two steel washers (washers for 1/2 inch bolts are about the right size and mass).
- Two 1/4 inch × 6 inch steel rods
- 4 rubber bands (#33, 1/8 × $3^1/_2$ inch)

Chapter 19 Student Text Objectives

- Identify a cycle of harmonic motion.
- Recognize common oscillators.
- Know the relationship between period and frequency.
- Understand how to identify and measure amplitude.
- Recognize the difference between linear motion and harmonic motion graphs.
- Interpret graphs of harmonic motion.
- Determine amplitude and period from a harmonic motion graph.
- Recognize when two oscillators are in phase or out of phase.
- Understand the role of restoring force in how oscillators work.
- Learn the relationship between amplitude and period for a pendulum.
- Recognize simple oscillators.

Skill and Practice Sheets

- 19.1 Period and Frequency
- 19.2 Harmonic Motion Graphs

Other Resources

- *Math Sheets*: Reciprocals and Negative One as an Exponent
- *Color Teaching Tools*: Period and Frequency, Harmonic Motion Graphs, Amplitude, Phase of Harmonic Motion

Introduction

Harmonic motion - motion that repeats in cycles.

Examples of harmonic motion:
-pendulum
-swing
-rotation of Earth
-Earth revolving around the sun

Cycle - a unit of motion that repeats over and over.

Earth's orbital cycle around the sun is one year.

What comes to mind when you think of a person being hypnotized?

Students may respond by citing examples of hypnosis being performed in movies or caroons that they have seen. The desired response is that students will state that the person performing the hypnosis, swings an object like a pocket watch, back and forth until the person is under the "spell" of the hypnotist.

Let's take a moment to examine the motion of the pocket watch.

It is a good idea to have a pocket watch or similar object in the classroom so that you can demonstrate this motion to students. Walk around the classroom swinging the pocket watch so that each student is able to see the pattern of its motion.

How is this motion different from the types of motion that we have already learned about?

The pocket watch is moving back and forth. The types of motion that we have already studied involved motion along a line or a curve.

Very good. Now, let me call your attention to this globe.

If you don't have a globe in your classroom, borrow one from a history teacher. Spin the globe around slowly so that each student has a clear view of its motion.

How would you describe the motion of this globe?

The globe moves around and around.

That is correct! We have now witnessed that objects move in many different ways. Some objects go from place to place along a curve or line, while others go around and around or back and forth. The two new types of motion that we have discovered are examples of harmonic motion.

Write the words, *harmonic motion*, on the chalkboard.

Harmonic motion is motion that repeats in cycles. Many important systems in nature and many useful inventions rely upon harmonic motion.

Retrieve the globe and repeat its round and round motion. While slowly spinning the globe, ask students the following question:

Can anyone describe how harmonic motion affects systems in nature like the seasons or phases of the moon?

The seasons and phases of the moon are caused by the Earth's harmonic motion. The Earth-sun system has an orbital cycle of one year, which creates the seasons. The observed phases of the moon are created by the monthly orbital cycle of the Earth-moon system.

In today's investigation, you will observe and describe the motion of a pendulum, a system in which a mass swings back and forth on a string. You will also learn how to recognize and describe other examples of harmonic motion.

1 Make a pendulum

Amplitude - the maximum amount the pendulum swings away from its resting position.

Gather the materials that are needed to complete the investigation. Refer to your investigation handout for the proper setup of the pendulum.

Walk around to ensure that students have set up the pendulum correctly.

Start the pendulum swinging and watch it for a minute. Think about how to describe the motion.

Allow students time to think about describing the motion. In the meantime, write the words *cycle* and *amplitude* on the chalkboard. Review the meanings of both *cycle* and *amplitude* with students. Demonstrate how to measure amplitude by tracing the path of the angle formed as the pendulum moves away from the center.

Complete steps a-c in part 1 of the investigation.

Harmonic Motion

19A

Question for this Investigation

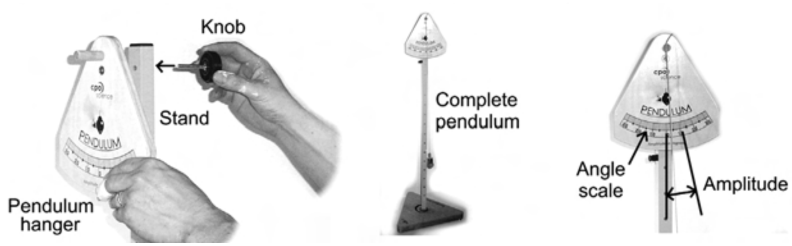
How does physics describe the repeating motion of a pendulum or swing?

Harmonic motion is motion that repeats in cycles. Many important systems in nature and many useful inventions rely on harmonic motion. For example, the phases of the moon and the seasons are caused by Earth's harmonic motion. This Investigation will explore harmonic motion using a pendulum. The concepts you learn with the pendulum will also apply to other examples of harmonic motion.

Materials

- Physics stand
- Graph paper
- Timer and 1 photogate
- Pendulum apparatus

1 Make a pendulum



Start the pendulum swinging and watch it for a minute. Think about how to describe the motion.

a. Write one sentence about the motion using the word "cycle."

b. The *amplitude* is the maximum amount the pendulum swings away from its resting position. The resting position is straight down. One way to measure amplitude is the angle the pendulum moves away from center. Write one sentence describing the motion of your pendulum using the word "amplitude."

c. Draw a sequence of sketches that describe one complete cycle using arrows to indicate the direction the pendulum is going at that point in the cycle.

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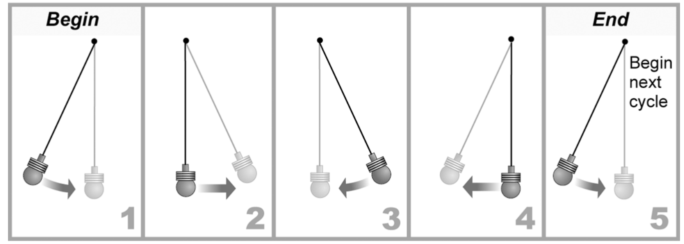
1a. The pendulum completes a cycle when released by traveling past its resting point and then returning to the point at which it was released.

1b. When the pendulum is released, it passes the resting position and travels to the same amplitude on the other side before returning to the point at which it was released.

1c. Drawings of one cycle:



2 3 4 **(shown on next page)**

2a.

Table 1: Pendulum period data: Time for 10 cycles (sec)

Trial 1	Trial 2	Trial 3	Average
17.53	17.47	17.56	17.52
		Period (avg /10)	1.75

2c. I started the stopwatch when I released the pendulum and stopped the stopwatch after having counted ten complete cycles.

3. There are no questions to answer in part 3.

4a. The time measurement from the Timer is 0.8783 s.

4b. This is not the period of the pendulum. The Timer is timing the time that has elapsed between when the pendulum passes through the resting position and when it returns to the resting position again after having traveled to one side. This is not a complete cycle; it is one half of a cycle.

4c. The time measured by the stopwatch is one half the period since it is measuring the time for the pendulum to complete one half of a cycle. If the time measured by the timer is multiplied by 2, the result should be approximately equal to the period of the pendulum, and it is: 0.8783 x 2 = 1.76 s, which is very close to the period measured in part 2, 1.75 s.

2 Oscillators and period

Oscillator - a system with harmonic motion.

Period - the time it takes for an oscillator to complete one cycle.

A fan oscillates ten cycles in 34.25 seconds. What is the period of the oscillating fan?

1. Count out ten cycles of the fan.
2. Use a stopwatch to measure the time needed to complete ten cycles.
3. Divide this time by ten.

34.25 seconds ÷ 10 = 3.425 seconds.

Add the terms, *oscillator* and *period*, to the list of vocabulary terms on the chalkboard.

In this part of the investigation, we will observe an example of another system that demonstrates harmonic motion. This system is called an *oscillator.* Suppose I made the following statement: “As Lillian tried to decide what she should wear on the first day of school, she oscillated between the green tee and the blue blouse”. Who can tell me what the term, *oscillate* means in this instance?

In this statement, oscillate means to be indecisive or to go back and forth about a choice that must be made.

Good. So if we describe the motion of an object as oscillating, how do you think the object moves?

An oscillating object swings or moves back and forth.

Let’s list some examples of oscillators.

Students will offer a variety of responses, for example, oscillating fans, pendulums from grandfather clocks, and vibrating guitar strings. Accept all reasonable responses.

The period of the oscillator is the time it takes to complete each cycle. You can determine the period of an oscillator by following a few simple steps.

Write each of the following numbered steps on the chalkboard.

1. Count out ten cycles of the oscillator.
2. Use a stopwatch to measure the time needed to complete ten cycles.
3. Divide this time by ten to get the time required to complete one cycle. This is the period of the oscillator.

Let’s try a sample calculation. A fan oscillates ten cycles in 34.25 seconds. What is the period of the oscillating fan?

Write the information from the sample calculation on the chalkboard.

Steps 1 and 2 have already been done for you. Now all that remains is to divide the time needed to complete 10 cycles by ten.

The period is 34.25 seconds ÷ 10 = 3.425 seconds.

Read and complete part 2 of the investigation. You will collect data from three trials. Measure the length of the string used to affix the pendulum. It will be useful to you in the next part of the investigation. Use your stopwatch to determine the time needed for the pendulum to complete 10 cycles. Record the time from each trial in Table 1. Once you have completed three trials, find the average of the data.

Remind students that the average can be found by adding together each of the recorded times and dividing by the number of trials, which in this instance is three.

Record the period of the pendulum in the space provided on Table 1.

3 Measuring period with a photogate

In this section of the investigation, we will use a photogate to measure the period of the pendulum. Use the diagrams in section 3 to help you setup your photogate. Try to keep the string length close to the length that you used in part 2 of the investigation.

Walk around the classroom to ensure that students have the proper setup.

Complete steps 1-4.

Allow time for students to complete this task.

4 Thinking about what you observed

Use the information gathered to answer questions a-c. You may find it helpful to refer to your results from part 2 of the investigation.

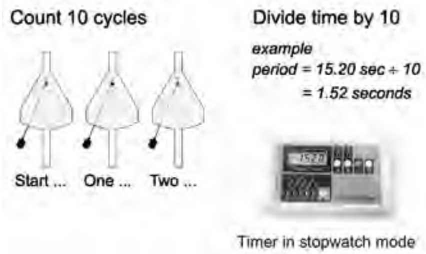
Discuss the students’ answers to the questions. Clarify any uncertainties before moving on to part 4.

2 Oscillators and period

a. Use the stopwatch to measure the period of your pendulum. Time ten cycles Do three trials and use Table 1 to record your data.

b. Divide the average time for ten cycles by 10 to get the period.

c. Write a one sentence description of how you measured the period.

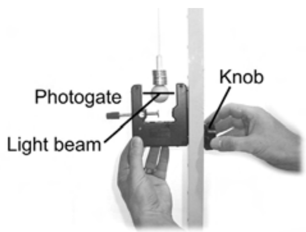


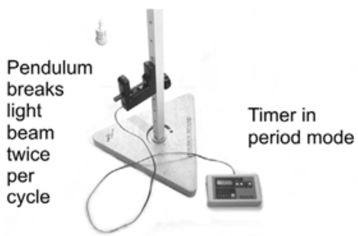
19A

Table 1: Pendulum period data: Time for 10 cycles (sec)

Trial 1	Trial 2	Trial 3	Average
Period of pendulum (average divided by10)			

3 Measuring period with a photogate





1. Attach the photogate as shown in the diagram. The pendulum breaks the light beam when it swings through the photogate. Try to keep the string length close to the length you used in part 2.
2. Put the Timer in period mode and let the pendulum swing through the light beam.
3. The reset (O) button does two things. If you press reset once the display freezes allowing you to write down a number before it changes. Pressing reset a second time starts another measurement.

4 Thinking about what you observed

a. Write down the time measurement you get from the Timer.

b. Is the time you get from the Timer the period of the pendulum? Explain why the time is or is not the period of the pendulum (hint: compare to your results from part 2).

c. Explain how the time measured by the Timer is related to the period of the pendulum.

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5 (shown on next page)

5a. Describing experiments:
1. Keep the length of the pendulum and the mass constant, measure the period as the amplitude is changed by changing the angle through which pendulum is moved away from the resting position.
2. Keep the amplitude and the length of the pendulum constant, measure the period as the mass is changed by adding washers.
3. Keep the amplitude and the mass the same, measure the period as the length of the pendulum is changed by varying the length of the string.

5b. Experimental data

Effect of Amplitude on Period
(Length = 82cm Mass = 10 washers)

Amplitude (degrees)	Trial 1 Period(s)	Trial 2 Period(s)	Average Period(s)
10	1.755	1.755	1.755
20	1.764	1.764	1.764
30	1.778	1.776	1.777

Effect of Mass on Period
(Length = 82 cm Amplitude = 20 degrees)

Mass (# washers)	Trial 1 Period(s)	Trial 2 Period(s)	Average Period(s)
10	1.764	1.764	1.764
6	1.769	1.769	1.769
3	1.777	1.775	1.776

Effect of Length of String on Period
(Amp = 20 deg Mass = 3 wash)

Length (cm)	Trial 1 Period(s)	Trial 2 Period(s)	Average Period(s)
82	1.777	1.775	1.764
60	1.513	1.513	1.513
30	1.044	1.046	1.045

5 What variables affect the period of the pendulum?

Three variables are:
-amplitude
-mass of the pendulum
-length of the string

You can change several things about the pendulum. Do you think that all changes made to the pendulum will affect its period?

Some changes will have an affect on the period of the pendulum, while other changes may not.

The three variables that are easiest to change are the amplitude, the mass, and the length of the string. Look at the diagrams in section 4 of the investigation. Do you think that altering any of the variables will affect the period of the pendulum?

Allow students to formulate hypotheses about the affect of changing each of the variables. Ask students to offer statements that support their hypotheses.

Think of three different experiments you can do to see which of the variables, if any, affect the period of the pendulum. Write down one sentence describing each experiment. This is step a of part 4 of the investigation. Complete steps b and c in part 4 of the investigation. Use the example in step c as a guide for explaining your findings.

Allow time for students to create and carry out different experiments.

Were any of your hypotheses about changing any of the variables correct?

Encourage students to participate in discussions about their hypotheses: both correct and incorrect. For those which were incorrect, ask students to explain how the data collected helped them to question their hypothesis, and arrive at a different conclusion. This is also a great opportunity to remind students that formulating and testing hypotheses is an important part of the scientific method.

Follow the instructions in step d to describe what happens to the amplitude and the period as the pendulum swings. Remember to let the pendulum swing naturally.

6 Sketch a graph of the motion using amplitude and period

We can use graphs to help us understand harmonic motion. Everyone should now focus on part 5 of the investigation. The most common type of graph used to express harmonic motion plots amplitude, on the y-axis, versus time, on the x-axis. Notice that each axis on the graph is labeled with proper units. Which unit of measurement is used to express amplitude for this graph?

The amplitude is measured in degrees.

Which unit of measurement is used for time?

The time is measured in seconds.

Good answer.

Point out both the x- and y-axes and the units to students so that they can see exactly where each is located on the investigation handout.

Why do the words, *left* and *right* appear along the y-axis?

The pendulum swings back and forth. The words left and right represent the back and forth motion of the pendulum.

Now, follow the steps in the diagram to sketch a graph showing the motion of the pendulum.

Circulate among your students to ensure that each student is performing the task correctly.

Has everyone plotted amplitude and drawn a smooth curve through the points?

Allow time for students to complete their graphs.

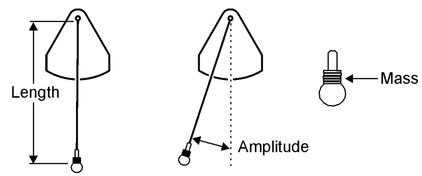
Use the information from your graph to answer questions a-d.

Review the students’ responses.

5 What variables affect the period of a pendulum?

You can change several things about the pendulum.

- The amplitude (angle)
- The mass (add or subtract washers)
- The length of the string (measure as shown)



a. Think of three experiments you can do to see what variables affect the period of the pendulum. Write down one sentence describing each experiment.

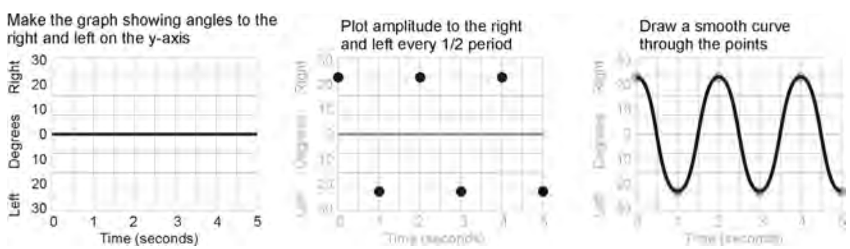
b. Do the three experiments and record the measurements you make to assess the effect of changing each variable.

c. Write a sentence about the effect of each variable. Write a second sentence explaining how the data you took support the statement you made about each variable. For example,

"We found that changing _____ had almost no effect on the period. We know because when we changed ____ from ____ to ____ the period only changed from ____ to ____, which is a very small difference.

6 Sketch a graph of the motion using amplitude and period

The most common type of graph puts amplitude on the vertical (*y*) axis and time on the horizontal (*x*) axis. Follow the steps in the diagram below to sketch a graph showing the motion of the pendulum.



a. How many complete cycles does your graph show?

b. Describe how to determine the amplitude of motion from a harmonic motion graph.

c. Describe how to determine the period from a harmonic motion graph.

d. What is the amplitude and period of the motion shown on the graph in the diagram above?

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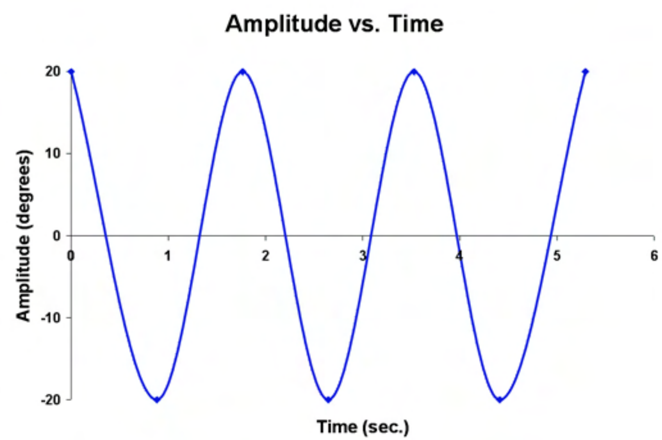
5 6

5c. Conclusions drawn from data:
1. We found that changing the amplitude had almost no effect on the period. We know because when we changed the amplitude from 10 to 20 and then 30 degrees, the period only changed from 1.755 s to 1.764 s to 1.777 s, which is a very small difference.
2. We found that changing the mass of the pendulum had almost no effect on the period. We know because when we changed the number of washers from 10 to 6 to 3, the period only changed from 1.764 s to 1.769 s to 1.776 s, which is a very small difference.
3. We found that changing the length of the pendulum did have an effect on the period. We know because when we changed the length from 82 cm to 60 cm to 30 cm, the period changed from 1.764 s to 1.513 s to 1.045 s, which are significant differences.

6a. This graph shows three complete cycles.



6b. The amplitude can be determined by looking at the degree measurement of the peak of the graph (either left or right).

6c. The period can be determined by looking at the time difference between two left peaks or two right peaks on the graph.

6d. Amplitude = 20 degrees; Period = 2 s.

Introduction

Newton's second law gives the relationship between force, mass, and motion.

Examples of objects that exhibit harmonic motion when disturbed:
-swing
-guitar string

Natural frequency - the frequency at which an object tends to move in harmonic motion.

What happens to an object if it is disturbed?

When an object is disturbed it will move.

Good answer. You have already learned that force applied to a mass results in motion. Newton's second law gives the relationship between motion, force, and mass. While the second law still applies, there are new results as the second law is applied to systems in harmonic motion. Consider what happens when a child sits on a swing and is pushed. How would you describe the resulting motion?

Prompt the class to describe the back and forth motion of the child on the swing. Encourage responses that lead the class to the idea that a force continues to pull the swing back to its starting point.

Very good. The child on the swing is in harmonic motion. The starting position is the equilibrium position. As the child is pushed forward there is a force that pulls the child back toward equilibrium. This is an example of stable equilibrium because it leads to harmonic motion. Many objects exhibit harmonic motion when disturbed. For example, when you pluck a guitar string or drop a pencil on the floor, the both vibrate. If you pluck the guitar string ten times consecutively, it vibrates at the same frequency each time. Why do you think this is so?

Get students involved in this discussion as they will arrive at some interesting but important ideas.

The frequency at which objects tend to move in harmonic motion is called the natural frequency. Everything that can oscillate, from a pencil dropped on the floor to a plucked guitar string, has a natural frequency. Most oscillators have more than one. This investigation will explore the idea of natural frequency, and how the natural frequency of a system is related to Newton's laws.

1 Make an oscillator

To make an oscillator you need:

1. Mass to provide inertia;
2. Restoring force connected to the mass

Restoring force - any force that tends to return a system to equilibrium.

Let's begin the investigation by making an oscillator. In order to make an oscillator you need to start with two objects. First, you must have a mass that provides inertia. You also need a restoring force connected to the mass. A restoring force is any force that tends to return a system to equilibrium. Think about the pendulum for a moment. How do you think the restoring force on the pendulum is created?

Set up a pendulum in front of the class. Discuss that gravity combined with the constraint of the string create the restoring force.

What is the restoring force on the pendulum at its lowest point?

Let the pendulum hang motionless. At this point it is in equilibrium and there are no forces acting to pull it one way or the other.

Watch as I push the pendulum a little to one side. There is a force that pulls it toward the center. Why does it keep going when it gets to the center where there is no force acting on it anymore?

Lead the class until students arrive at the conclusion that inertia keeps the pendulum going through the center.

With mechanical systems like a pendulum, harmonic motion comes from the action of forces and inertia. Recall that a restoring force always tries to pull a system back toward equilibrium. For example, gravity always pulls the pendulum toward the center no matter which side it is on. Because it has inertia, the pendulum passes right through the middle and keeps going. The restoring force then slows it and accelerates the pendulum back toward equilibrium. However, the pendulum overshoots again and goes too far. The cycle repeats over and over again to create harmonic motion.

Demonstrate this to students as they watch the pendulum swing back and forth.

Follow each step in part 1 of the investigation to make an oscillator using two rubber bands and a washer. Use the diagrams shown in part 1 to help you. Be sure to stretch your rubber bands a few times to "break them in."

Walk around the classroom and observe as students construct the oscillator.

Natural Frequency

19B

Question for this Investigation:

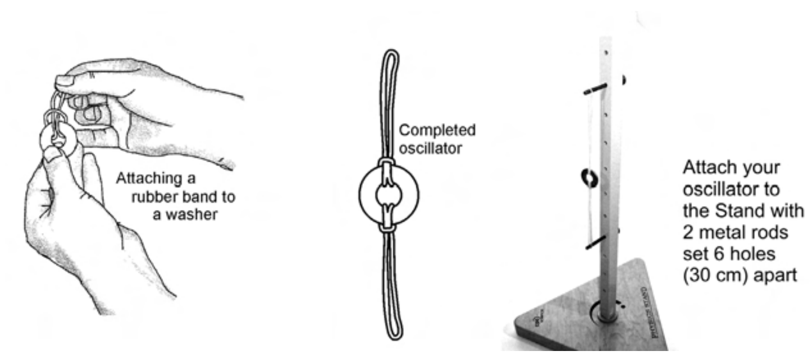
How do force and mass affect harmonic motion?

Newton's second law gives the relationship between motion, force, and mass. While the second law still applies, there are new effects that result when the second law is applied to systems in harmonic motion. This Investigation will explore the idea of natural frequency, and how the natural frequency of a system is related to Newton's laws.

Materials

- Physics stand
- Two 1/4" × 6" steel rods
- Timer and 1 photogate
- 4 rubber bands (#33, 1/8 × 3 1/2")
- Two steel washers. Washers for 1/2" bolts are about the right size and mass.

1 Make an oscillator



1. Fix two rubber bands onto opposite sides of a washer as shown in the diagram.
2. Stretch the rubber bands a few times to "break them in."
3. Set the two steel rods into the stand so they are 6 holes apart. The washer is suspended by the stretched rubber bands.
4. Pull the washer about 1 centimeter to one side. Release the washer and observe its motion.

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1

There are no questions to answer in part 1.

Where do you find harmonic motion?

Sound is a traveling vibration of air molecules. Musical instruments and stereo speakers are oscillators that we design to create sounds with certain cycles that we enjoy hearing. When a stereo is playing, the speaker cone moves back and forth rapidly. The cyclic back-and-forth motion pushes and pulls on air, creating tiny oscillations in pressure. The pressure oscillations travel to your eardrum and cause it to vibrate. Vibrations of the eardrum move tiny bones in the ear setting up more vibrations that are transmitted by nerves to the brain. There is harmonic motion at every step of the way, from the musical instrument's performance to the perception of sound by your brain.

Light is the result of harmonic motion of the electric and magnetic fields. The colors that you see in a picture come from the vibration of electrons in the molecules of paint. Each color of paint contains different molecules that oscillate with different cycles to create the different colors of light you see.

Almost all modern communication technology relies on harmonic motion. The electronic technology in a cell phone uses an oscillator that makes more than 100 million cycles each second. When you tune into a station at 101 on the FM dial, you are actually setting the oscillator in your radio to 101,000,000 cycles per second.

2 Measure the natural frequency

Frequency = 1/period

Natural frequency obtained from the number of cycles per second is given in units of hertz (Hz).

Oscillators have a natural frequency, the frequency at which the oscillator tends to oscillate, or move in harmonic motion. You can use a photogate to measure the natural frequency by measuring the period of your oscillator. How are period and frequency related?

Students should recall that frequency is equal to 1 ÷ period.

Good answer. Because period and frequency are related, if you know one you know the other. However, measuring the natural frequency is tricky. You have to hold the photogate so the washer breaks the light beam just once per cycle.

Demonstrate the proper way to position the photogate. To ensure that students correctly set up the apparatus, work with students and take a step-by-step approach to the procedure.

Nudge the rubber bands around until the washer oscillates back and forth without twisting much. Practice your technique until you can get the washer to oscillate back and forth with an amplitude of about one centimeter.

Allow students time to practice the technique. Use this time to also review the meaning of amplitude, the maximum distance an object moves from its equilibrium position.

Hold a photogate near the edge of the oscillating washer so the washer breaks the light beam once per cycle. You should see the light on the photogate flash red and green as the washer breaks the beam.

Confirm that students observed the flashing red and green lights on the photogate before moving on.

Set the Timer to measure the period of the oscillation. Complete three trials. Record the measurement for each trial in Table 1. Find the average period, then convert it to natural frequency. What will be the unit of measurement for the natural frequency?

Natural frequency, obtained from the number of cycles per second, is measured in hertz (Hz).

3 Thinking about what you observed

How would you expect the natural frequency of your oscillator to be affected by the strength of the rubber bands?

What would happen to the natural frequency if you made the washer heavier?

Reflect on what you observed in part 2 of the investigation as you respond to each of the questions (a-d). I'd like to hear your thoughts so we are going to discuss your answers as a class.

These questions are great for a class discussion. Encourage students to share their observations and to participate in the dialogue.

Does the washer and rubber band oscillator have a natural frequency? How do you know?

Students should observe that the washer and rubber band oscillator displays harmonic motion and therefore has a natural frequency. Accept all reasonable explanations supporting the oscillator's natural frequency.

Who would like to describe how you measured the natural frequency?

Solicit responses from a number of investigation teams. Steer students toward explanations describing how the washer moved relative to the photogate.

Suppose you strengthen the rubber bands. How would you expect the natural frequency of the oscillator to be affected? Do you think it would get higher, lower, or stay about the same?

Allow students to speculate about the result of strengthening the rubber bands. Do not offer any concrete answers just yet regarding the effect of increasing force. Students will discover more about this as the investigation moves forward. Record students' hypotheses on the chalkboard and tell them that the class will revisit them later to determine whether or not they were correct.

What would happen to the natural frequency if you made the washer heavier by adding more mass? Would the frequency increase, decrease, or stay about the same?

As with question 3c, encourage discussion among students. Although some students may arrive at a correct response, wait until the end of the investigation to confirm their conjecture.

2 Measure the natural frequency

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You can use a photogate to measure the natural frequency by measuring the period of your oscillator. Remember, period and frequency are related (*frequency* = 1 ÷ *period*).

The measurement is tricky to do. You have to hold the photogate so the washer breaks the light beam just once per cycle.

1. Nudge the rubber bands around until the washer oscillates back and forth without twisting much. Practice your technique until you can get the washer to oscillate back and forth with an amplitude of about 1 centimeter
2. Hold a photogate near the edge of the oscillating washer so the washer breaks the light beam once per cycle. You should see the light on the photogate flash red and green as the washer breaks the beam.
3. Set the Timer to measure the period of the oscillation. Record three measurements and take the average.
4. Convert period to frequency.

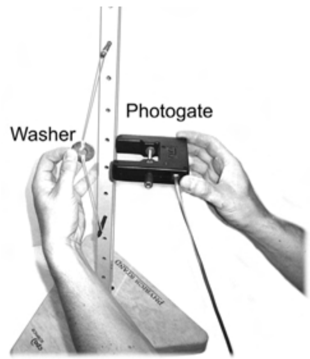

Table 1: Natural frequency data (Hz) for 1 washer with 2 rubber bands

Trial 1 Period (sec)	Trial 2 Period (sec)	Trial 3 Period (sec)	Average Period (sec)
		Natural Frequency (Hz)	

3 Thinking about what you observed

a. Does the washer-and-rubber-band oscillator have a natural frequency? How do you know?

b. Explain in one or two sentences how you measured the natural frequency. (Hint: describe the washer's motion relative to the photogate.)

c. What would happen to the natural frequency if you made the rubber bands stronger? Would the natural frequency get higher, lower, or stay about the same? Explain your reasoning in a few sentences?

d. What would happen to the natural frequency if you made the washer heavier by adding more mass? Would the natural frequency get higher, lower, or stay about the same? Explain your reasoning in a few sentences?

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2 3

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Table 1: Natural Frequency Data (Hz) for 1 washer with 2 rubber bands

Trial 1 Period (sec)	Trial 2 Period (sec)	Trial 2 Period (sec)	Average Period (sec)
.0902	.897	.0901	.0900
		Natural Frequency (Hz)	11.1

3a. Yes, the washer-and-rubber band oscillator has a natural frequency. Each time the oscillator was displaced, the frequency was about the same (1/.0900 sec. = 11.1 Hz).

3b. To find the natural frequency I nudged the washer and rubber band so the washer would oscillate back and forth. As it oscillated, it broke the beam of the photogate and I was able to measure the period of its oscillation. I calculated the natural frequency from the average period measured in three trials.

3c. I think the natural frequency would get higher if the rubber bands were stronger. This would happen because the rubber band would be able to pull the washer back toward the middle with more force which would accelerate the washer faster. The effect on the washer would be that it oscillates faster, giving the washer a higher natural frequency.

3d. If the washer was heavier, the natural frequency would be lower because there would be more mass to move back and forth. The rubber band would still provide the same amount of force, but a washer with more mass would be accelerated less, resulting in a slower oscillation and lower natural frequency.

4 Changing the force in your oscillator

Look at the oscillator shown in part 4 of the investigation. You can see that it has four rubber bands and one washer. Build the oscillator. The rubber bands should be snug on the washer and in opposite pairs. Before making any measurements, stretch the rubber bands to the same distance as in part 2. Do you think the natural frequency of the 4-band oscillator will differ from the first oscillator you constructed?

Allow students to express their opinions. Students may expect a different natural frequency, but may not be sure of the exact reasoning. This is okay as they will gather more support for the differences in the natural frequency as the investigation progresses.

Measure the natural frequency of the oscillator.

After students have measured the natural frequency of the oscillator, lead them in a discussion about the factors that affected its frequency. Guide them toward understanding the impact of adding additional rubber bands and its subsequent effect on the oscillator's natural frequency.

5 How the natural frequency depends on mass and force

Both mass and force affect the natural frequency of an oscillator. In this part of the investigation, you will create four different combinations of rubber bands and washers. Find the period of each combination. Refer to the procedure in part 2 of the investigation for help. Use those same techniques to measure the natural period of your oscillator for the four combinations shown in Table 2.

Review each rubber band-washer configuration with students. Remind students that the rubber bands should be stretched the same distance for each trial. This will lead to more consistent results.

Conduct three trials for each configuration of the rubber bands and washers. Find the average of the three trials and record your answers in table 2. Use the average period for each combination of rubber bands and washers to calculate the natural frequency.

Allow time for students to collect data.

6 Thinking about what you observed

Look at the data you recorded in Table 2. How do these results compare to the answers you gave to questions 3c and 3d? If your data does not support your responses to questions 3c and 3d, then provide new answers based on your new observations.

Allow time for students to compare their original responses to the questions (3c and 3d) and to make changes to those responses if necessary. Lead students in a discussion about changes they made and the reason(s) for those changes.

The natural frequency depends on the ratio of two variables: one from the rubber bands and one mostly from the steel washer(s). What are the two variables and how should they be arranged as a ratio?

Help students to see that the natural frequency of an oscillator is a balance between the strength of the restoring force and the mass providing the inertia. Rubber bands provide restoring forces while much of the inertia is from the washers. The natural frequency can be represented by the ratio of restoring force to mass. Write the ratio on the chalkboard (restoring force ÷ mass).

Suppose you wanted to increase the natural frequency of the oscillator. How could you accomplish this?

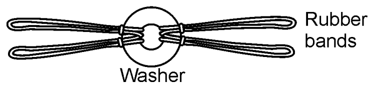
The natural frequency can be increased by reducing the mass or increasing the restoring force.

Very good! In the same way, if you wanted to decrease the natural frequency, either the mass must increase or the restoring force has to be decreased.

Emphasize the proportional relationship between restoring force and mass to students.

4 Changing the force in your oscillator

Completed 4-band oscillator



Rubber bands should be snug on washer and in opposite pairs. Pre-stretch the rubber bands before making any measurements.

1. Tie a second set of rubber bands around the washer as shown.
2. Measure the natural frequency with the rubber bands stretched the same distance as part 2.

5 How the natural frequency depends on mass and force

1. Use the techniques of part 2 to measure the natural period of your oscillator for the four combinations shown in Table 2.
2. To be consistent, the rubber bands should be stretched the same distance for all trials.

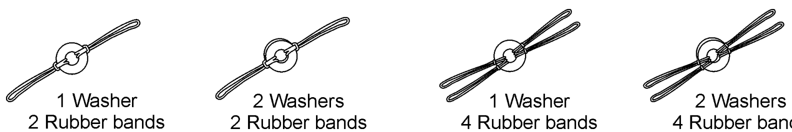


Table 2: Natural Period and Frequency Data (sec)

	2 Rubber bands 1 Washer	4 Rubber bands 1 Washer	2 Rubber bands 2 Washers	4 Rubber bands 2 Washers
Trial 1				
Trial 2				
Trial 3				
Average period				
Natural frequency (Hz)				

6 Thinking about what you observed

a. Did the results agree with your answers to questions 3d and 3e? If not, give new answers based on your observations.

b. The natural frequency depends on the ratio of two variables. One from the rubber bands and one mostly from the steel washer(s). What are the two variables and how should they be arranged as a ratio?

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4. There are no questions to answer in part 4.
5.

Table 2: Natural Period and Frequency Data (sec)

	2 rubber bands 1 washer	4 rubber bands 1 washer	2 rubber bands 2 washers	4 rubber bands 2 washers
Trial 1	0902	.0659	.1232	.0887
Trial 2	.0897	.0644	.1249	.0877
Trial 3	.0901	.0650	.1251	.0851
Avg Period (sec)	.0900	.0651	.1244	.0872
Nat Freq (Hz)	11.1	15.4	8.0	11.5

6a. Yes they agreed.

6b. The two variables involved with the natural frequency are the mass of the washer(s) and the force provided by the rubber bands. Natural frequency ~ force/mass